Nano-Optics

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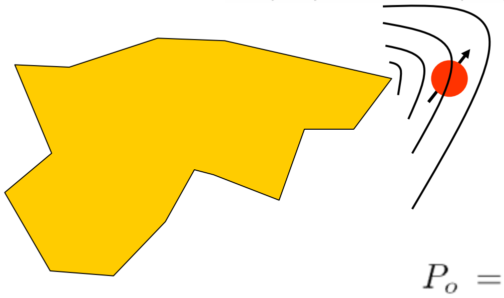
Dipole in inhomogeneous space

$$\mathbf{E}(\mathbf{r}_o) = \mathbf{E}_o(\mathbf{r}_o) + \mathbf{E}_s(\mathbf{r}_o)$$



$$P_o = \frac{|\boldsymbol{\mu}|^2}{12\pi} \frac{\omega}{\varepsilon_o \varepsilon} k^3$$

$P = P_0 + P_S$

$P/P_0 = 1 + P_S/P_0$

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Rate of energy dissipation in inhomogeneous space

$$\frac{P}{P_o} = 1 + \frac{6\pi\varepsilon_o\varepsilon}{|\boldsymbol{\mu}|^2} \frac{1}{k^3} \operatorname{Im}\{\boldsymbol{\mu}^* \cdot \mathbf{E}_s(\mathbf{r}_o)\}$$

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Another point of view

undriven harmonically oscillating dipole:

$$\frac{d^2}{dt^2}\boldsymbol{\mu}(t) + \gamma_o \frac{d}{dt}\boldsymbol{\mu}(t) + \omega_o^2 \boldsymbol{\mu}(t) = 0$$

$$\boldsymbol{\mu}(t) = \mathrm{Re}\left\{\boldsymbol{\mu}_o \, \mathrm{e}^{-i\omega_o\sqrt{1-(\gamma_o^2/4\omega_o^2)}\,t} \; \mathrm{e}^{-\gamma_o t/2}\right\}$$

damped oscillation with slight frequency shift

require that $\gamma_o \ll \omega_o$

then the average energy stored in the oscillator is:

$$\bar{W}(t) = \frac{m}{2q^2}\left[\omega_o^2\mu^2(t) + \dot{\mu}^2(t)\right] = \frac{m\omega_o^2}{2q^2}\,|\boldsymbol{\mu}_o|^2\,\mathrm{e}^{-\gamma_o t}$$

lifetime of the oscillator: $\tau_o = 1/\gamma_o$

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Classical decay rate

Energy conservation requires that the decrease in oscillator energy must equal the energy losses

quantum yield

$$\bar{W}(t\!=\!0) - \bar{W}(t) = q_i \int_0^t P_o(t')\,dt'$$

using $\bar{W}(t)$ as just defined and $P_o(t) = \frac{|\boldsymbol{\mu}(t)|^2}{4\pi\varepsilon_o}\frac{\omega_o^4}{3c^3}$

we get

$$\gamma_o = q_i \frac{1}{4\pi\varepsilon_o}\frac{2q^2\omega_o^2}{3\,m\,c^3}$$

classical expression for the atomic decay rate

At optical wavelengths a value for the decay rate of ~$2\cdot 10^{-8}\,\mathrm{s}^{-1}$ is obtained.

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Inhomogeneous environment

$$\frac{d^2}{dt^2}\boldsymbol{\mu}(t) + \gamma_o \frac{d}{dt}\boldsymbol{\mu}(t) + \omega_o^2 \boldsymbol{\mu}(t) = \frac{q^2}{m}\mathbf{E}_s(t)$$

$\mathbf{E}_s$ … secondary local field

Expectation: Interaction with $\mathbf{E}_S$ causes a shift in resonance frequency and a modification of the decay rate.

Trial solution for the dipole moment and the driving field:

$$\boldsymbol{\mu}(t) = \mathrm{Re}\left\{\boldsymbol{\mu}_o \, \mathrm{e}^{-i\omega t} \, \mathrm{e}^{-\gamma t/2}\right\}$$

$$\mathbf{E}_s(t) = \mathrm{Re}\left\{\mathbf{E}_o \, \mathrm{e}^{-i\omega t} \, \mathrm{e}^{-\gamma t/2}\right\}$$

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Change in the decay rate

assume again that $\gamma_o \ll \omega_o$ ➡ reject terms in γ^2

Interaction with the driving field $\mathbf{E}_s$ is weak

➡ $\frac{d^2}{dt^2}\boldsymbol{\mu}(t) + \gamma_o \frac{d}{dt}\boldsymbol{\mu}(t) + \omega_o^2 \boldsymbol{\mu}(t) = \frac{q^2}{m}\mathbf{E}_s(t)$

$$\omega_o^2 \boldsymbol{\mu}(t) \gg \frac{q^2}{m}\mathbf{E}_s(t)$$

➡ $$\frac{\gamma}{\gamma_o} = 1 + q_i \frac{6\pi\varepsilon_o}{|\boldsymbol{\mu}_o|^2}\frac{1}{k^3}\,\mathrm{Im}\{\boldsymbol{\mu}_o^* \cdot \mathbf{E}_s(\mathbf{r}_o)\}$$

using $\gamma_o = q_i \frac{1}{4\pi\varepsilon_o}\frac{2q^2\omega_o^2}{3\,m\,c^3}$

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compare to $$\frac{P}{P_o} = 1 + \frac{6\pi\varepsilon_o\varepsilon}{|\boldsymbol{\mu}|^2}\frac{1}{k^3}\,\mathrm{Im}\{\boldsymbol{\mu}^*\cdot\mathbf{E}_s(\mathbf{r}_o)\}$$

derived from energy conservation (evaluation of Poyntings theorem)

to $$\frac{\gamma}{\gamma_o} = 1 + q_i\,\frac{6\pi\varepsilon_o}{|\boldsymbol{\mu}_o|^2}\frac{1}{k^3}\,\mathrm{Im}\{\boldsymbol{\mu}_o^*\cdot\mathbf{E}_s(\mathbf{r}_o)\}$$

It follows that for $q_i = 1$

$$\frac{\gamma}{\gamma_o} = \frac{P}{P_o}$$

change in the decay rate is identical to the
change in the rate of energy dissipation

Frequency shift

$$\Delta\omega = \omega - \omega_o$$

$$\Delta\omega = \omega\left[1 - \sqrt{1 - \frac{1}{\omega^2}\left[\frac{q^2}{m\,|\boldsymbol{\mu}_o|^2}\,\mathrm{Re}\{\boldsymbol{\mu}_o^*\cdot\mathbf{E}_s\} + \frac{\gamma\gamma_o}{2} - \frac{\gamma\gamma}{4}\right]}\,\right]$$

expanding the square root to first order and
neglecting the quadratic terms in γ.

$$\frac{\Delta\omega}{\gamma_o} = q_i\,\frac{3\pi\varepsilon_o}{|\boldsymbol{\mu}_o|^2}\frac{1}{k^3}\,\mathrm{Re}\{\boldsymbol{\mu}_o^*\cdot\mathbf{E}_s\}$$

Small shift: same order of magnitude as the radiative linewidth

Maybe observable in high precision experiments.

Quantum yield

$$Q = \frac{\gamma_r}{\gamma_r + \gamma_{nr}}$$

γ_r and γ_{nr} … radiative and non-radiative decay rate

homogeneous environment: $Q = q_i$ intrinsic quantum yield

Now: γ_r and γ_{nr} are functions of the local environment

γ_{nr}: radiation absorbed in the environment

γ_r: radiation emitted to the farfield

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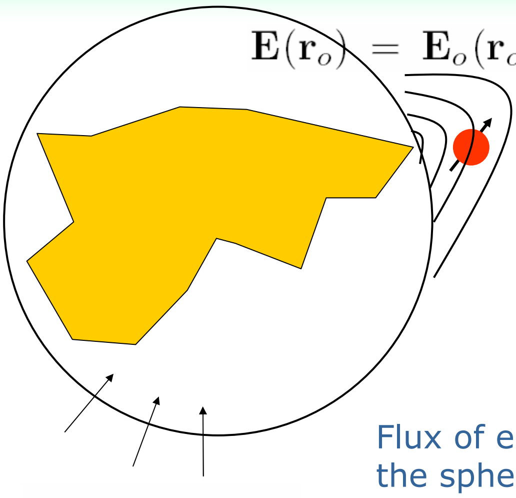
Non-radiative decay rate

$$\mathbf{E}(\mathbf{r}_o) = \mathbf{E}_o(\mathbf{r}_o) + \mathbf{E}_s(\mathbf{r}_o)$$



$$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$$

Flux of e.m. energy into the sphere must be equal to the dissipation rate

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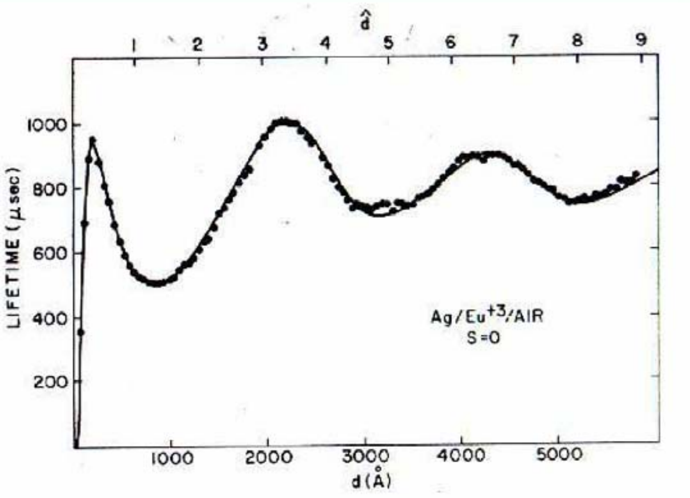
Experiment vs. theory

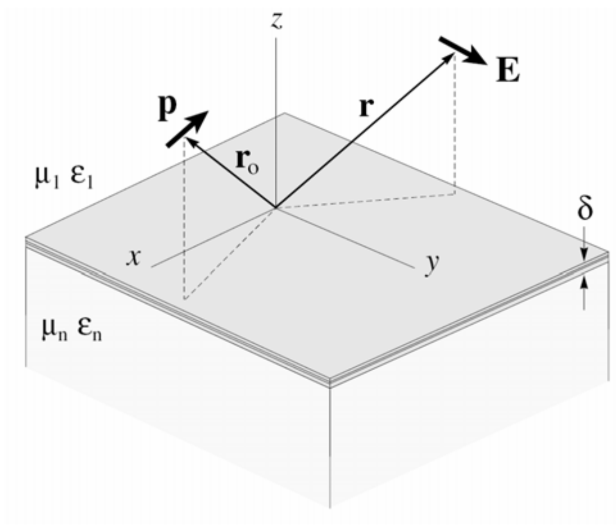


R. R. Chance, A. Prock, and R. Silbey, "Molecular fluorescence and energy transfer near interfaces," in *Advances in Chemical Physics* (I. Prigogine and S. A. Rice, eds.) **37**, 1–65, New York: Wiley (1978).

data by Drexhage

Dipole emission near planar interfaces

Setting



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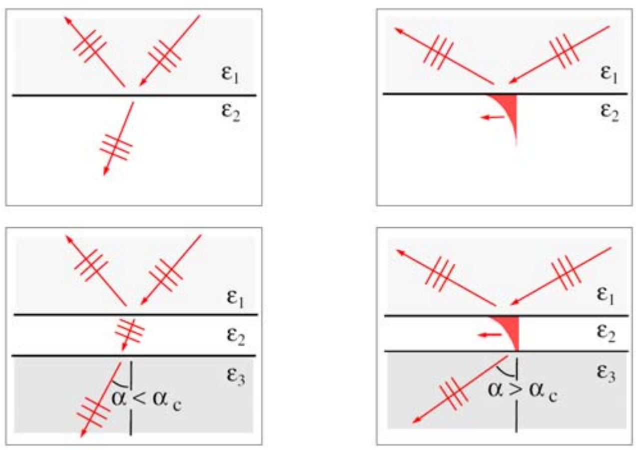
Allowed and forbidden light

allowed light | forbidden light



$$\varepsilon_3 > \varepsilon_1 > \varepsilon_2$$

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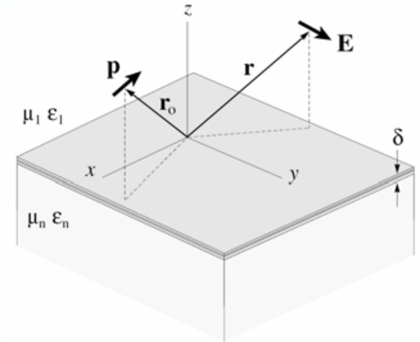
To solve



we will try to express the dipole field in terms of plane and evanescent waves for which we know how to reflect and refract them at the interface
also: make use of the dyadic Green´s function

no interface: $\mathbf{E}(\mathbf{r}) \;=\; \omega^2 \mu_o \mu_1 \,\overleftrightarrow{\mathbf{G}}_o\,(\mathbf{r},\mathbf{r}_o)\,\boldsymbol{\mu}$

we are looking for an angular spectrum representation of $\overleftrightarrow{\mathbf{G}}_o\,(\mathbf{r},\mathbf{r}_o)$

$$\left[\nabla^2 + k_1^2\right]\mathbf{A}(\mathbf{r}) \;=\; -\mu_o\mu_1\,\mathbf{j}(\mathbf{r})$$

$$\mathbf{j}(\mathbf{r}) \;=\; -i\omega\,\delta(\mathbf{r}-\mathbf{r}_o)\;\boldsymbol{\mu}$$

compare to the definition $\left[\nabla^2 + k^2\right] G_o(\mathbf{r},\mathbf{r}') \;=\; -\delta(\mathbf{r}-\mathbf{r}')$

$$\Rightarrow\quad \mathbf{A}(\mathbf{r}) \;=\; \boldsymbol{\mu}\,\frac{k_1^2}{i\omega\varepsilon_o\varepsilon_1}\,\frac{e^{ik_1|\mathbf{r}-\mathbf{r}_o|}}{4\pi|\mathbf{r}-\mathbf{r}_o|}$$

angular spectrum representation:

$$\mathbf{A}(\mathbf{r}) \;=\; \boldsymbol{\mu}\,\frac{k_1^2}{8\pi^2\omega\varepsilon_o\varepsilon_1}\iint\limits_{-\infty}^{\infty}\frac{1}{k_{z_1}}\,\mathrm{e}^{i\left[k_x(x-x_o)\,+\,k_y(y-y_o)\,+\,k_{z_1}|z-z_o|\right]}\,dk_x\,dk_y$$

Use $\mathbf{E} = i\omega[1 + k_1^{-2}\nabla\nabla\cdot]\mathbf{A}$

and compare to $\mathbf{E}(\mathbf{r}) \;=\; \omega^2\mu_o\mu_1\,\overleftrightarrow{\mathbf{G}}_o(\mathbf{r},\mathbf{r}_o)\,\boldsymbol{\mu}$

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$$\overleftrightarrow{\mathbf{G}}_o(\mathbf{r},\mathbf{r}_o) \;=\; \frac{i}{8\pi^2}\iint\limits_{-\infty}^{\infty}\overleftrightarrow{\mathbf{M}}\,\mathrm{e}^{i\left[k_x(x-x_o)\,+\,k_y(y-y_o)\,+\,k_{z_1}|z-z_o|\right]}\,dk_x\,dk_y$$

with

z<z$_o$ z>z$_o$

$$\overleftrightarrow{\mathbf{M}} \;=\; \frac{1}{k_1^2k_{z_1}}\begin{bmatrix} k_1^2-k_x^2 & -k_xk_y & \mp k_xk_{z_1} \\ -k_xk_y & k_1^2-k_y^2 & \mp k_yk_{z_1} \\ \mp k_xk_{z_1} & \mp k_yk_{z_1} & k_1^2-k_{z_1}^2 \end{bmatrix}$$

This is still the old Green´s function for the homogeneous space, however, in a plane wave expansion.

To apply the Fresnel reflection and transmission coefficients we need to divide the matrix in an s-polarized and a p-polarized part.

$$\overleftrightarrow{\mathbf{M}}(k_x,k_y) \;=\; \overleftrightarrow{\mathbf{M}}^{s}(k_x,k_y) \;+\; \overleftrightarrow{\mathbf{M}}^{p}(k_x,k_y)$$

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$$\overleftrightarrow{\mathbf{M}}^{s} = \frac{1}{k_{z_1}(k_x^2+k_y^2)} \begin{bmatrix} k_y^2 & -k_x k_y & 0 \\ -k_x k_y & k_x^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\overleftrightarrow{\mathbf{M}}^{p} = \frac{1}{k_1^2(k_x^2+k_y^2)} \begin{bmatrix} k_x^2 k_{z_1} & k_x k_y k_{z_1} & \mp k_x(k_x^2+k_y^2) \\ k_x k_y k_{z_1} & k_y^2 k_{z_1} & \mp k_y(k_x^2+k_y^2) \\ \mp k_x(k_x^2+k_y^2) & \mp k_y(k_x^2+k_y^2) & (k_x^2+k_y^2)^2/k_{z_1} \end{bmatrix}$$

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Fresnel coefficients

$$r^s(k_x, k_y) = \frac{\mu_2 k_{z_1} - \mu_1 k_{z_2}}{\mu_2 k_{z_1} + \mu_1 k_{z_2}}$$

$$t^s(k_x, k_y) = \frac{2\mu_2 k_{z_1}}{\mu_2 k_{z_1} + \mu_1 k_{z_2}}$$

$$k_{z_1} = \sqrt{k_1^2 - (k_x^2 + k_y^2)}$$

$$k_{z_2} = \sqrt{k_2^2 - (k_x^2 + k_y^2)}$$

$$r^p(k_x, k_y) = \frac{\varepsilon_2 k_{z_1} - \varepsilon_1 k_{z_2}}{\varepsilon_2 k_{z_1} + \varepsilon_1 k_{z_2}}$$

$$t^p(k_x, k_y) = \frac{2\varepsilon_2 k_{z_1}}{\varepsilon_2 k_{z_1} + \varepsilon_1 k_{z_2}} \sqrt{\frac{\mu_2 \varepsilon_1}{\mu_1 \varepsilon_2}}$$

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Green´s function upper halfspace

dyadic Green's function of the reflected field

$$\overleftrightarrow{\mathbf{G}}_{ref}(\mathbf{r},\mathbf{r}_o) = \frac{i}{8\pi^2}\iint_{-\infty}^{\infty}\left[\overleftrightarrow{\mathbf{M}}^{s}_{ref}+\overleftrightarrow{\mathbf{M}}^{p}_{ref}\right] e^{i\left[k_x(x-x_o)+k_y(y-y_o)+k_{z_1}(z+z_o)\right]}\, dk_x dk_y$$

$$\overleftrightarrow{\mathbf{M}}^{s}_{ref} = \frac{r^s(k_x,k_y)}{k_{z_1}(k_x^2+k_y^2)}\begin{bmatrix} k_y^2 & -k_xk_y & 0 \\ -k_xk_y & k_x^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

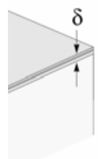
$$\overleftrightarrow{\mathbf{M}}^{p}_{ref} = \frac{-r^p(k_x,k_y)}{k_1^2(k_x^2+k_y^2)}\begin{bmatrix} k_x^2k_{z_1} & k_xk_yk_{z_1} & k_x(k_x^2+k_y^2) \\ k_xk_yk_{z_1} & k_y^2k_{z_1} & k_y(k_x^2+k_y^2) \\ -k_x(k_x^2+k_y^2) & -k_y(k_x^2+k_y^2) & -(k_x^2+k_y^2)^2/k_{z_1} \end{bmatrix}$$

$$\Rightarrow \quad \mathbf{E}(\mathbf{r}) = \omega^2\mu_o\mu_1\left[\overleftrightarrow{\mathbf{G}}_o(\mathbf{r},\mathbf{r}_o)+\overleftrightarrow{\mathbf{G}}_{ref}(\mathbf{r},\mathbf{r}_o)\right]\boldsymbol{\mu}$$

new Green´s function of upper halfspace!

Green´s function lower halfspace

$$\overleftrightarrow{\mathbf{G}}_{tr}(\mathbf{r},\mathbf{r}_o) = \frac{i}{8\pi^2}\iint_{-\infty}^{\infty}\left[\overleftrightarrow{\mathbf{M}}^{s}_{tr}+\overleftrightarrow{\mathbf{M}}^{p}_{tr}\right] e^{i\left[k_x(x-x_o)+k_y(y-y_o)-k_{z_n}(z+\delta)+k_{z_1}z_o\right]}\, dk_x dk_y$$



$$\overleftrightarrow{\mathbf{M}}^{s}_{tr} = \frac{t^s(k_x,k_y)}{k_{z_1}(k_x^2+k_y^2)}\begin{bmatrix} k_y^2 & -k_xk_y & 0 \\ -k_xk_y & k_x^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\overleftrightarrow{\mathbf{M}}^{p}_{tr} = \frac{t^p(k_x,k_y)}{k_1k_n(k_x^2+k_y^2)}\begin{bmatrix} k_x^2k_{z_n} & k_xk_yk_{z_n} & k_x(k_x^2+k_y^2)k_{z_n}/k_{z_1} \\ k_xk_yk_{z_n} & k_y^2k_{z_n} & k_y(k_x^2+k_y^2)k_{z_n}/k_{z_1} \\ k_x(k_x^2+k_y^2) & k_y(k_x^2+k_y^2) & (k_x^2+k_y^2)^2/k_{z_1} \end{bmatrix}$$

$$\mathbf{E}(\mathbf{r}) = \omega^2\mu_o\mu_1\,\overleftrightarrow{\mathbf{G}}_{tr}(\mathbf{r},\mathbf{r}_o)\,\boldsymbol{\mu}$$

new Green´s function of lower halfspace!

More than one interface

generalized Fresnel coefficients

for example: single layer with thickness d

$$r^{(p,s)} = \frac{r_{1,2}^{(p,s)} + r_{2,3}^{(p,s)}\,\exp(2ik_{2z}d)}{1 + r_{1,2}^{(p,s)} r_{2,3}^{(p,s)}\,\exp(2ik_{2z}d)}$$

$$t^{(p,s)} = \frac{t_{1,2}^{(p,s)} t_{2,3}^{(p,s)}\,\exp(ik_{2z}d)}{1 + r_{1,2}^{(p,s)} r_{2,3}^{(p,s)}\,\exp(2ik_{2z}d)}$$

$r_{i,j}^{(p,s)}$ $t_{i,j}^{(p,s)}$ reflection and transmission coefficients for the single interface (i, j).

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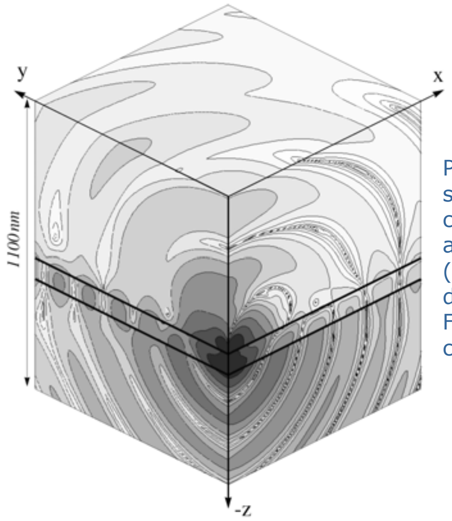
Example



Power density of a dipole above a slab waveguide depicted at a certain time. The dipole is located at h=20nm and its axis is in the (x, z) plane. θ=60°, λ=488nm, d=80nm, ε_1=1, ε_2=5, ε_3=2.25. Factor of 2 between successive contour lines.

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spontaneous decay rate

$$\frac{P}{P_o} = 1 + \frac{6\pi\varepsilon_o\varepsilon}{|\boldsymbol{\mu}|^2}\frac{1}{k^3}\,\mathrm{Im}\{\boldsymbol{\mu}^*\cdot\mathbf{E}_s(\mathbf{r}_o)\}$$

$$\frac{\gamma}{\gamma_o} = 1 + q_i\frac{6\pi\varepsilon_o}{|\boldsymbol{\mu}_o|^2}\frac{1}{k^3}\,\mathrm{Im}\{\boldsymbol{\mu}_o^*\cdot\mathbf{E}_s(\mathbf{r}_o)\}$$

identical for $q_i = 1$

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spontaneous decay rate

$\mathbf{E}_s(\mathbf{r}_o)$ corresponds to the reflected field

$$\mathbf{E}_{ref}(\mathbf{r}_o) = \omega^2\mu_o\mu_1\,\overleftrightarrow{\mathbf{G}}_{ref}(\mathbf{r}_o,\mathbf{r}_o)\,\boldsymbol{\mu}$$

$$\overleftrightarrow{\mathbf{G}}_{ref}(\mathbf{r},\mathbf{r}_o) = \frac{i}{8\pi^2}\iint_{-\infty}^{\infty}\left[\overleftrightarrow{\mathbf{M}}^{s}_{ref}+\overleftrightarrow{\mathbf{M}}^{p}_{ref}\right]e^{i\left[k_x(x-x_o)+k_y(y-y_o)+k_{z_1}(z+z_o)\right]}\,dk_x dk_y$$

with transformations

$$k_x = k_\rho\cos\phi,\quad k_y = k_\rho\sin\phi,\quad dk_x dk_y = k_\rho dk_\rho d\phi$$

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$$\overleftrightarrow{\mathbf{G}}_{ref}(\mathbf{r}_o,\mathbf{r}_o) = \frac{i}{8\pi k_1^2}\int_0^\infty \frac{k_\rho}{k_{z_1}}\begin{bmatrix} k_1^2 r^s - k_{z_1}^2 r^p & 0 & 0 \\ 0 & k_1^2 r^s - k_{z_1}^2 r^p & 0 \\ 0 & 0 & 2k_\rho^2 r^p \end{bmatrix} \mathrm{e}^{2ik_{z_1}z_o}\, dk_\rho$$

Integral over Φ is solved analytically

$$\Rightarrow \mathbf{E}_{ref}(\mathbf{r}_o) = \omega^2 \mu_o \mu_1 \overleftrightarrow{\mathbf{G}}_{ref}(\mathbf{r}_o,\mathbf{r}_o)\,\boldsymbol{\mu}$$

$$\Rightarrow \frac{P}{P_o} = 1 + \frac{6\pi\varepsilon_o\varepsilon}{|\boldsymbol{\mu}|^2}\frac{1}{k^3}\,\mathrm{Im}\{\boldsymbol{\mu}^*\cdot\mathbf{E}_s(\mathbf{r}_o)\}$$

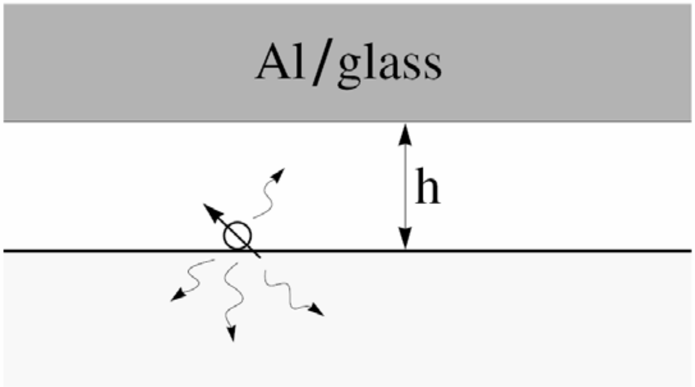
$$\frac{P}{P_o} = 1 + \frac{\mu_x^2+\mu_y^2}{p^2}\frac{3}{4}\int_0^\infty \mathrm{Re}\left\{\frac{s}{s_z}\left[r^s - s_z^2 r^p\right]\, \mathrm{e}^{2ik_1 z_o s_z}\right\} ds + \frac{\mu_z^2}{p^2}\frac{3}{2}\int_0^\infty \mathrm{Re}\left\{\frac{s^3}{s_z}\, r^p\, \mathrm{e}^{2ik_1 z_o s_z}\right\} ds$$

$s = k_\rho / k_1$

$\sqrt{1-s^2} = k_{z_1}/k_1$

$s_z = (1-s^2)^{1/2}$

Example



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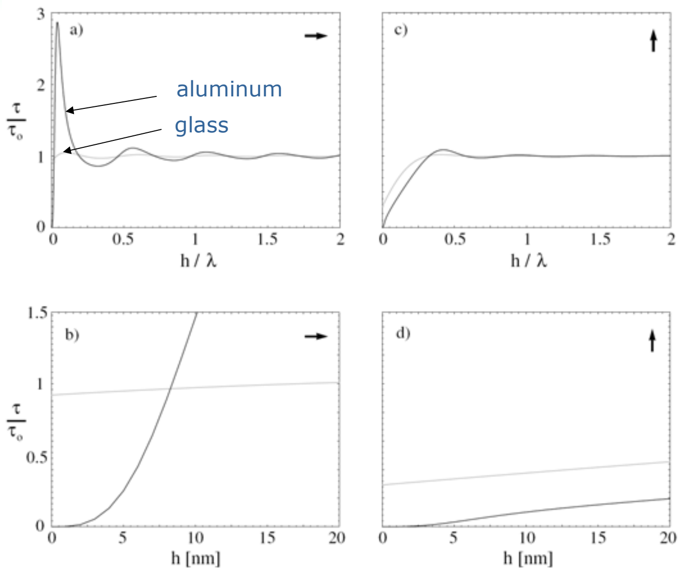

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Far fields

One of the advantages of using the angular spectrum representation is the straightforward derivation of the far field:

$$\mathbf{E}_{\infty}(s_x, s_y, s_z) \;=\; -ik\, s_z\, \hat{\mathbf{E}}(ks_x, ks_y; 0)\, \frac{e^{ikr}}{r}$$

$$\mathbf{s} \;=\; (s_x, s_y, s_z) \;=\; (\frac{x}{r}, \frac{y}{r}, \frac{z}{r})$$

different optical properties in the upper and lower half space ➡

$$\mathbf{s} \;=\; \begin{cases} (\frac{k_x}{k_1}, \frac{k_y}{k_1}, \frac{k_{z_1}}{k_1}) & z > 0 \\ (\frac{k_x}{k_n}, \frac{k_y}{k_n}, \frac{k_{z_n}}{k_n}) & z < 0 \end{cases}$$

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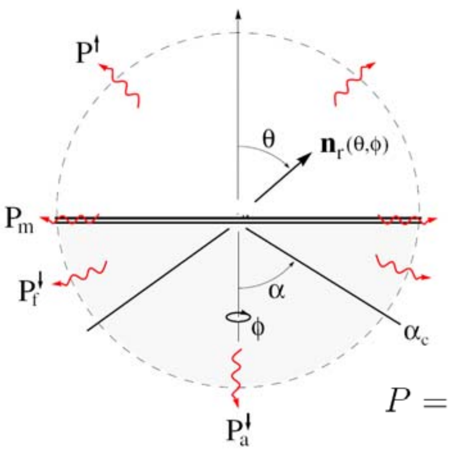
spherical vector components $\mathbf{E} = (E_r, E_\theta, E_\phi)$

upper half space $s_z = k_{z_1}/k_1 = \cos\theta$

lower half space $s_z = k_{z_n}/k_n = -\cos\theta$



$$P = P^{\uparrow} + P_a^{\downarrow} + P_f^{\downarrow} + P_m + P_i$$

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$$\mathbf{E} = \begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \frac{k_1^2}{4\pi\varepsilon_o\varepsilon_1} \frac{\exp(ik_j r)}{r}$$

$$\times \begin{bmatrix} [\mu_x \cos\phi + \mu_y \sin\phi] \cos\theta \, \Phi_j^{(2)} - \mu_z \sin\theta \, \Phi_j^{(1)} \\ -[\mu_x \sin\phi - \mu_y \cos\phi] \, \Phi_j^{(3)} \end{bmatrix}$$

$j \in [1, n]$ for upper or lower half space

$$\tilde{s}_z = \frac{k_{z_1}}{k_n} = \sqrt{(n_1/n_n)^2 - (s_x^2 + s_y^2)} = \sqrt{(n_1/n_n)^2 - \sin^2\theta}$$

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height of the dipole above the interface

$$\Phi_1^{(1)} = \left[e^{-ik_1 z_o \cos\theta} + r^p(\theta) \, e^{ik_1 z_o \cos\theta} \right]$$

dipole in a depth z_o below the interface

$$\Phi_1^{(2)} = \left[e^{-ik_1 z_o \cos\theta} - r^p(\theta) \, e^{ik_1 z_o \cos\theta} \right]$$

$$\Phi_1^{(3)} = \left[e^{-ik_1 z_o \cos\theta} + r^s(\theta) \, e^{ik_1 z_o \cos\theta} \right]$$

$$\Phi_n^{(1)} = \frac{n_n}{n_1} \frac{\cos\theta}{\tilde{s}_z(\theta)} \, t^p(\theta) \, e^{ik_n [z_o \tilde{s}_z(\theta) + \delta \cos\theta]}$$

$$\Phi_n^{(2)} = -\frac{n_n}{n_1} t^p(\theta) \, e^{ik_n [z_o \tilde{s}_z(\theta) + \delta \cos\theta]}$$

$$\Phi_n^{(3)} = \frac{\cos\theta}{\tilde{s}_z(\theta)} \, t^s(\theta) \, e^{ik_n [z_o \tilde{s}_z(\theta) + \delta \cos\theta]} .$$

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In the farfield, the magnetic field vector is transverse to the electric field vector and the time-averaged Poynting vector is calculated as

$$\langle \mathbf{S} \rangle = \frac{1}{2} \mathrm{Re}\{\mathbf{E} \times \mathbf{H}^*\} = \frac{1}{2} \sqrt{\frac{\varepsilon_o \varepsilon_j}{\mu_o \mu_j}} \, \mathbf{E} \cdot \mathbf{E}^* \, \mathbf{n}_r$$

radiated power per unit solid angle $d\Omega = \sin\theta \, d\theta \, d\phi$

$$P = p(\Omega) \, d\Omega = r^2 \, \langle \mathbf{S} \rangle \cdot \mathbf{n}_r$$

with $\boldsymbol{p(\Omega) = p(\theta, \phi)}$

radiation pattern

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Normalized radiation pattern

$$\frac{p(\theta,\phi)}{P_o} = \frac{3}{8\pi} \frac{\varepsilon_j}{\varepsilon_1} \frac{n_1}{n_j} \frac{1}{|\boldsymbol{\mu}|^2} \Big[\mu_z^2 \sin^2\theta \left|\Phi_j^{(1)}\right|^2 + [\mu_x \cos\phi + \mu_y \sin\phi]^2 \cos^2\theta \left|\Phi_j^{(2)}\right|^2 + [\mu_x \sin\phi - \mu_y \cos\phi]^2 \left|\Phi_j^{(3)}\right|^2 - \mu_z [\mu_x \cos\phi + \mu_y \sin\phi] \cos\theta \, \sin\theta \left[\Phi_j^{*(1)} \Phi_j^{(2)} + \Phi_j^{(1)} \Phi_j^{*(2)} \right] \Big]$$

p-polarized contribution of the vertical orientation

p-polarized contributions of the horizontal orientation

s-polarized contributions of the horizontal orientation

interferences between the p-polarized terms of the two major orientations

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Discussion

$$\left[\Phi_j^{*(1)}\,\Phi_j^{(2)} + \Phi_j^{(1)}\,\Phi_j^{*(2)}\right] \propto \left|t^{(p)}(\theta)\right|^2 \mathrm{e}^{-2\,z_o\,\mathrm{Im}\{\tilde{s}_z(\theta)\}}\ \mathrm{Re}\left\{\frac{\cos\theta}{\tilde{s}_z(\theta)}\right\}$$

vanishes in the forbidden zone where

$$\tilde{s}_z = \frac{k_{z_1}}{k_n} = \sqrt{(n_1/n_n)^2 - (s_x^2 + s_y^2)} = \sqrt{(n_1/n_n)^2 - \sin^2\theta}$$

becomes imaginary

→ no interferences → no dependence on ϕ

Furthermore,

$$\tilde{s}_z = \frac{k_{z_1}}{k_n} = \sqrt{(n_1/n_n)^2 - (s_x^2 + s_y^2)} = \sqrt{(n_1/n_n)^2 - \sin^2\theta}$$

being imaginary leads to a z-dependence of the pattern in the forbidden zone

Example

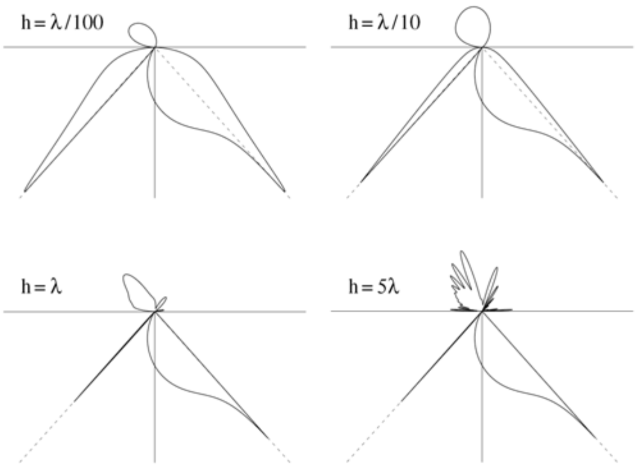

Figure 10.7: Radiation patterns of a dipole with orientation $\vartheta = 60°$ approaching a planar waveguide. $\lambda = 488\,nm$, $\delta = 80\,nm$, $\varepsilon_1 = 1$, $\varepsilon_2 = 5$, $\varepsilon_3 = 2.25$. The different heights $z_o = h$ of the dipole are indicated in the figure. The radiation patterns are shown in the plane defined by the dipole axis and the z-axis. Note that the *allowed light* does not depend on h and that the *forbidden light* is always symmetrical with respect to the vertical axis.

Experiment