



Nano-Optics

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Possible configurations: Overview

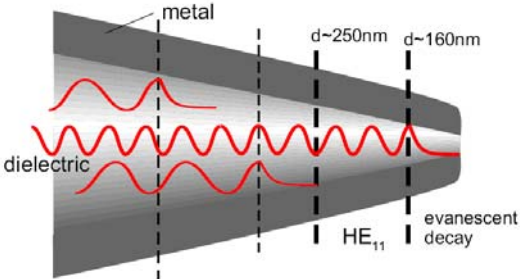
Illumination	near-field	near-field	far-field	far-field
Detection	near-field	far-field	near-field	far-field
Setups				

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Near-field illumination – far-field detection

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 Light propagation



metal

dielectric


$d \sim 250 \text{ nm}$ $d \sim 160 \text{ nm}$

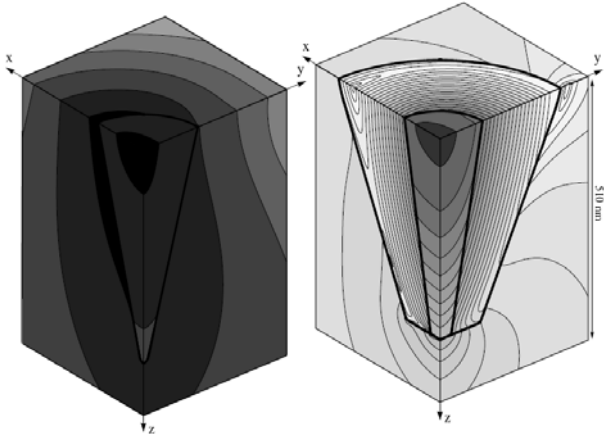
HE_{11} evanescent decay

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Contour lines of constant $|E|^2$ on three perpendicular planes through a dielectric probe (left) and an aperture probe (right). Factor of 4 between successive lines. $\lambda = 488 \text{ nm}$, $\epsilon_{\text{core}} = 2.16$, $\epsilon_{\text{coat}} = -34.5 + i 8.5$. The exciting HE_{11} mode is polarized along the x direction.

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Possible configurations: Overview

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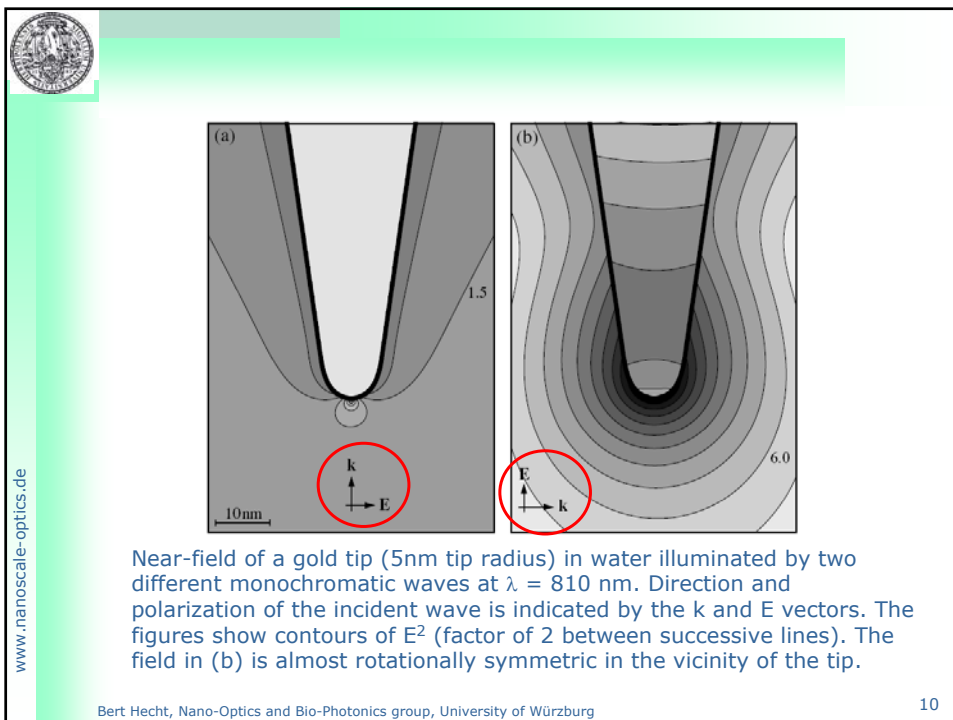
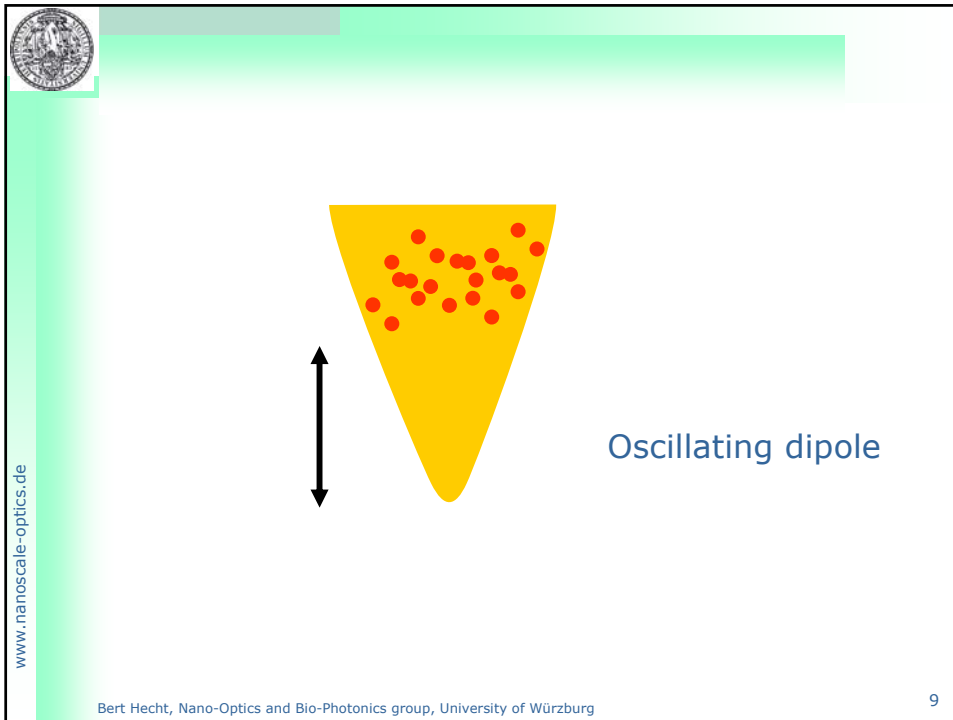
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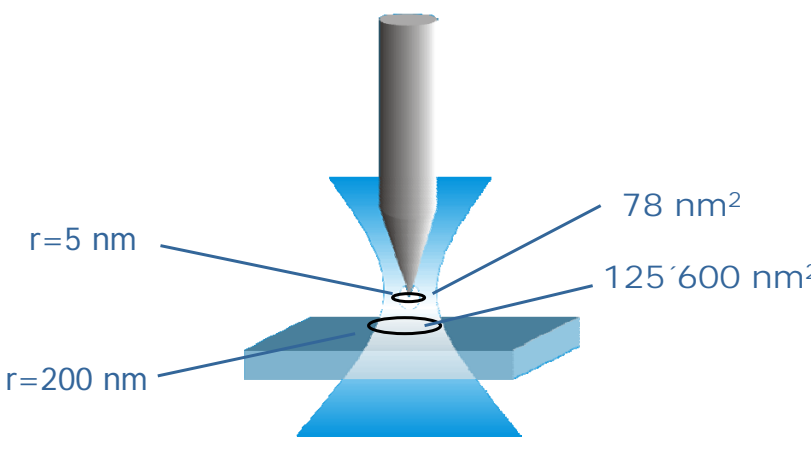
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 **Tip-enhanced microscopy**




$r=5 \text{ nm}$
 $r=200 \text{ nm}$
 78 nm^2
 $125\,600 \text{ nm}^2$

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 **Signal-to-background**

- optical interaction that is based on a n -th order nonlinear process
- assume that only the sample surface is active

Then the **far-field background** will be

$$S_{ff} \sim A I_o^n$$

A ... illuminated surface area
 I_o ... laser intensity

near-field signal

$$S_{nf} \sim a (f I_o)^n$$

f ... enhancement factor for the electric field intensity (E^2)
 a ... reduced area given by the tip size

require that $S_{nf}/S_{ff} > 1$ and assume $a = \pi(5\text{nm})^2$, $A = \pi(200\text{nm})^2$


$$\Rightarrow f > \sqrt[n]{1600}$$

$n=1$	$\rightarrow f=1600$
$n=2$	$\rightarrow f=40$
...	

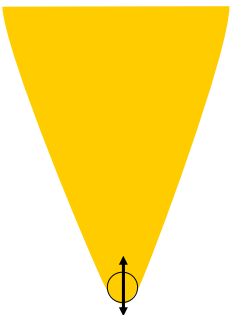
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$$\mathbf{p}(\omega) = \begin{bmatrix} \alpha_{\perp} & 0 & 0 \\ 0 & \alpha_{\perp} & 0 \\ 0 & 0 & \alpha_{\parallel} \end{bmatrix} \mathbf{E}_o(\omega)$$


$$\alpha_{\perp}(\omega) = 4\pi\epsilon_o r_o^3 \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 2}$$

$$\alpha_{\parallel}(\omega) = 2\pi\epsilon_o r_o^3 f_e(\omega)$$

$$\mathbf{E}(\mathbf{r}, \omega) = \mathbf{E}_o(\mathbf{r}, \omega) + \frac{1}{\epsilon_o} \frac{\omega^2}{c^2} \vec{\mathbf{G}}(\mathbf{r}, \mathbf{r}_o, \omega) \mathbf{p}(\omega)$$

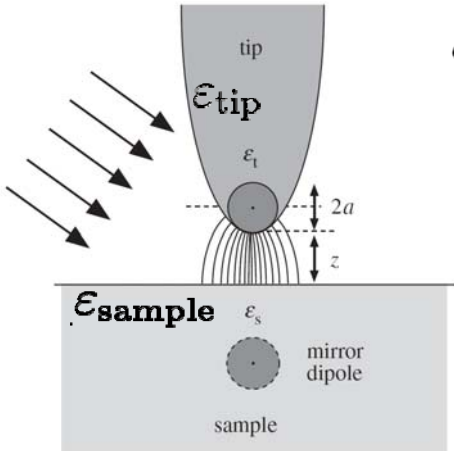
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Alternative signal extraction

by tip-sample gapwidth modulation



$$\alpha_{\text{eff}} = \frac{\alpha(1 + \beta)}{1 - \frac{\alpha\beta}{16\pi(a+z)^3}}$$

$$\alpha = 4\pi a^3 (\epsilon_{\text{tip}} - 1) / (\epsilon_{\text{tip}} + 2)$$

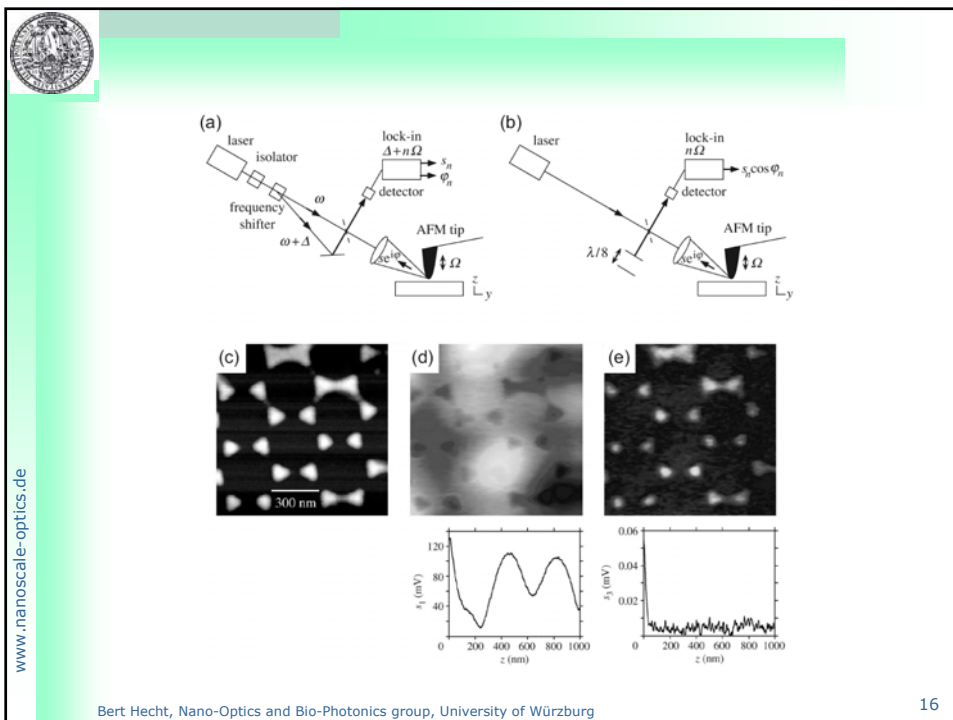
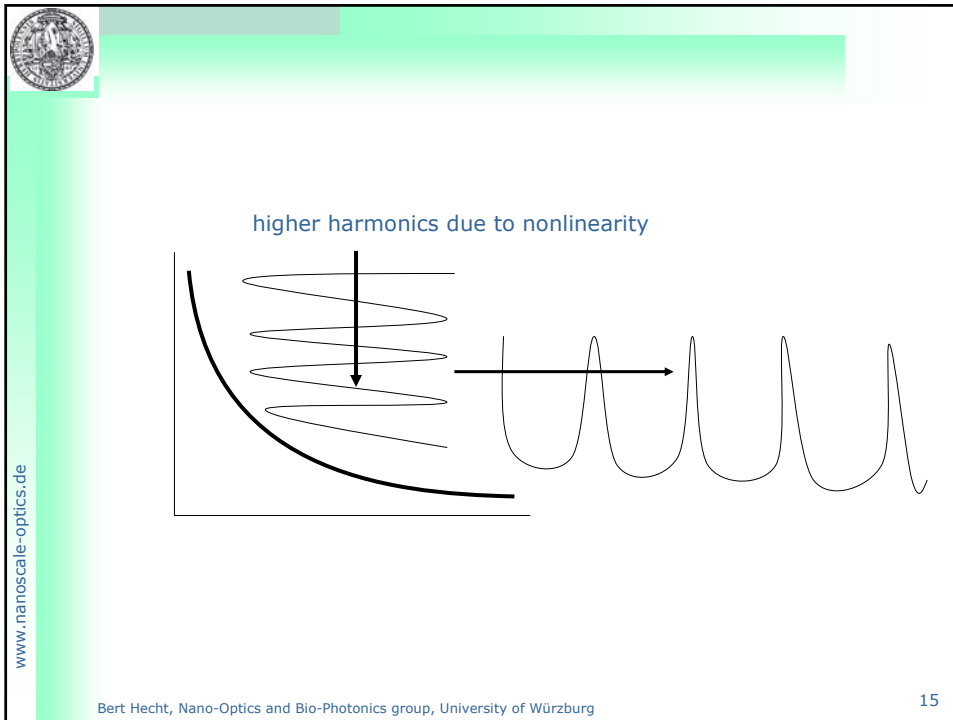
$$\beta = (\epsilon_{\text{sample}} - 1) / (\epsilon_{\text{sample}} + 1)$$

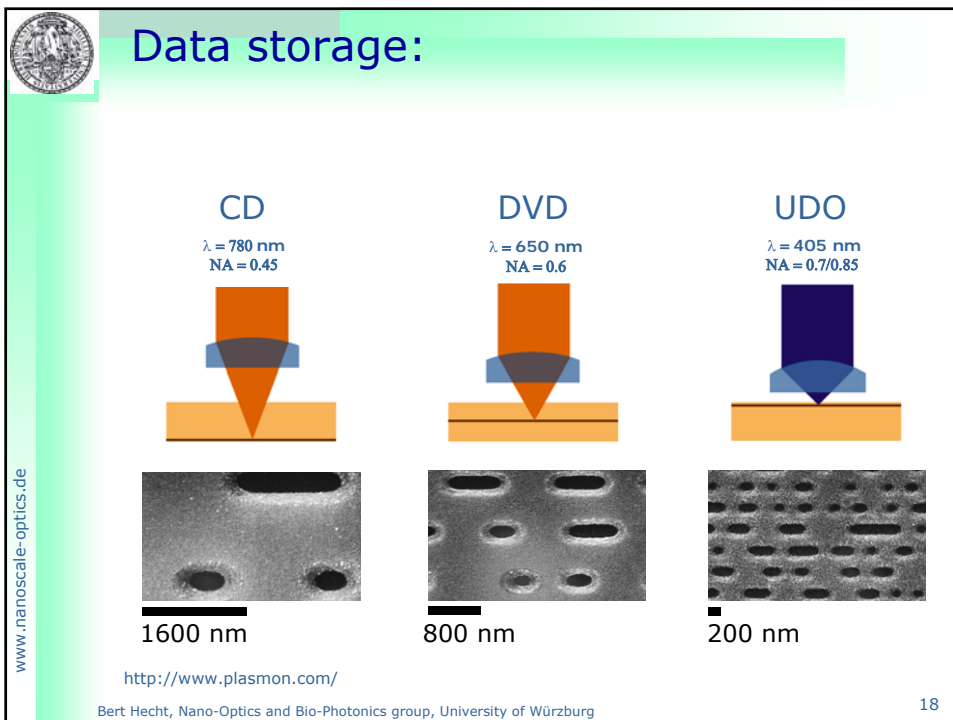
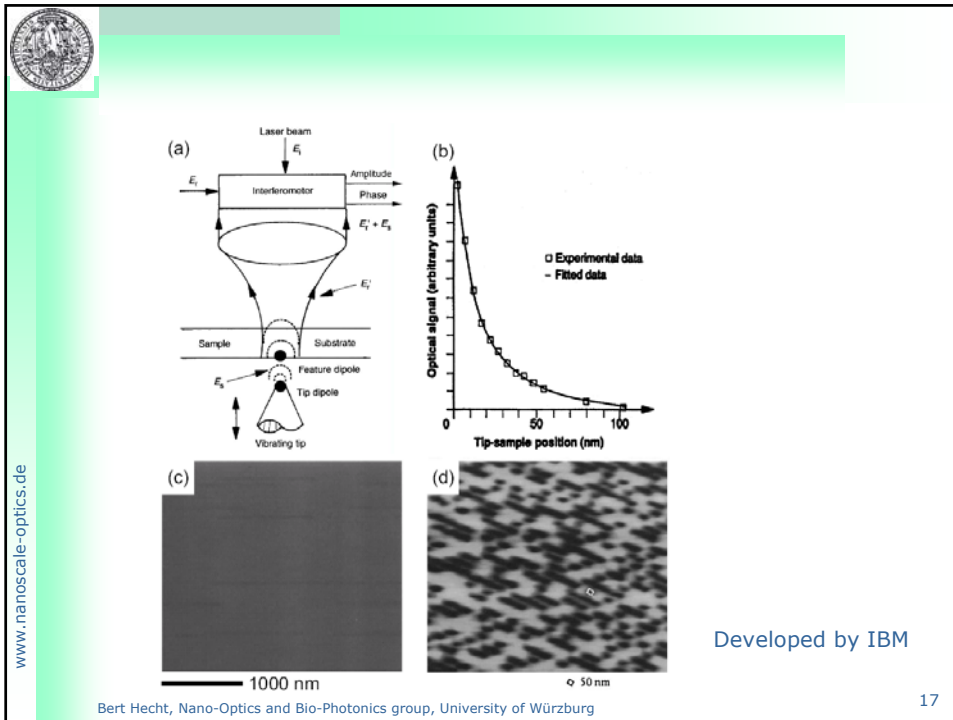
Scattering of the tip
Sample gap region
depends on ϵ_{sample}

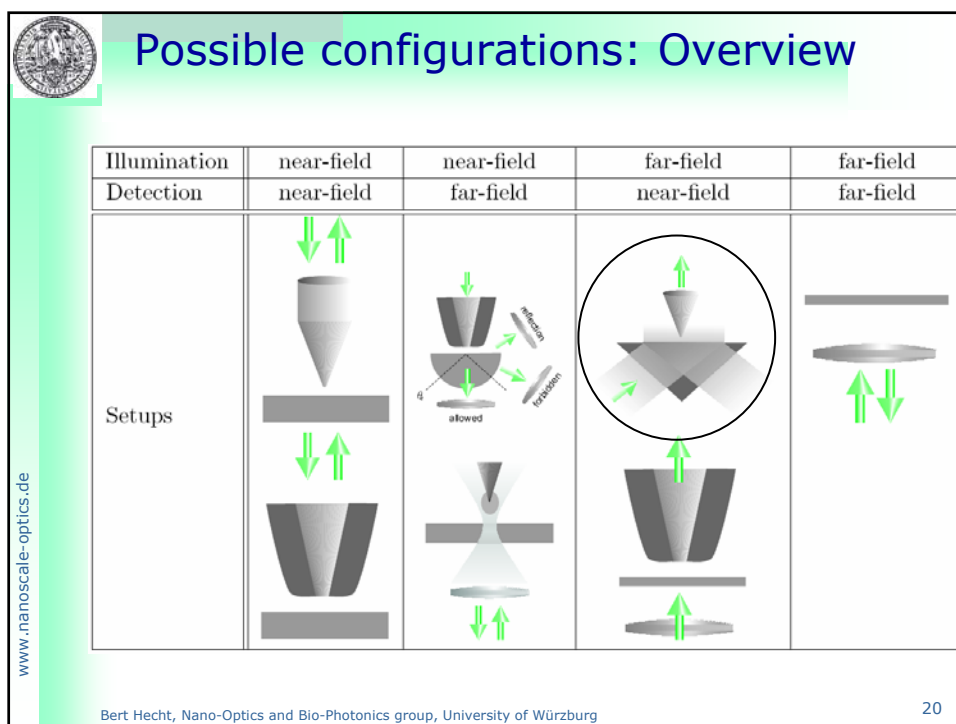
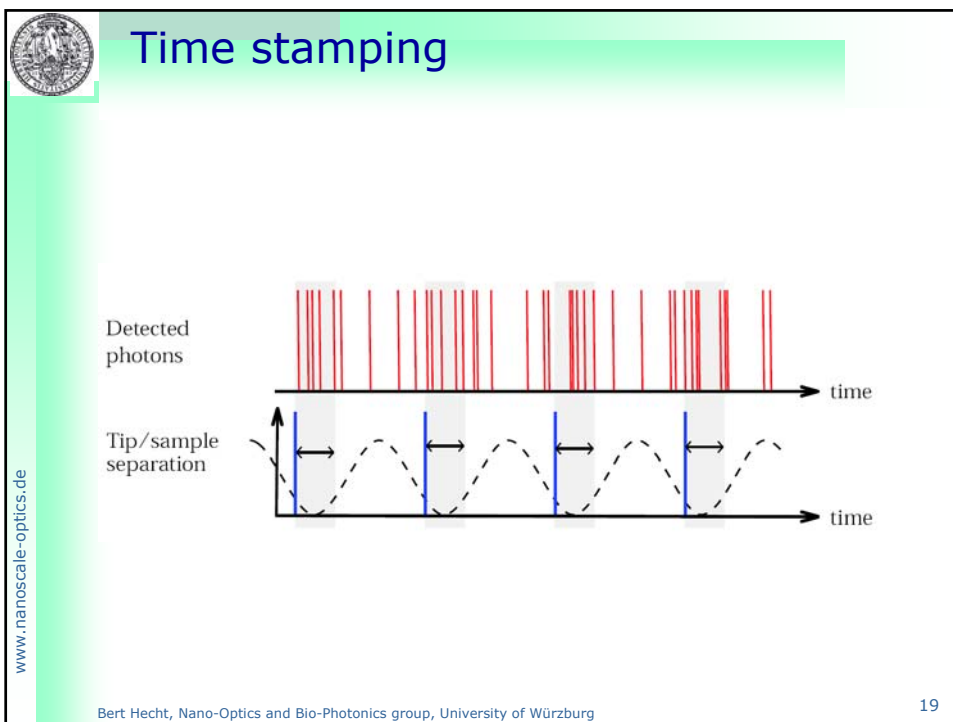
→ Contrast mechanism!


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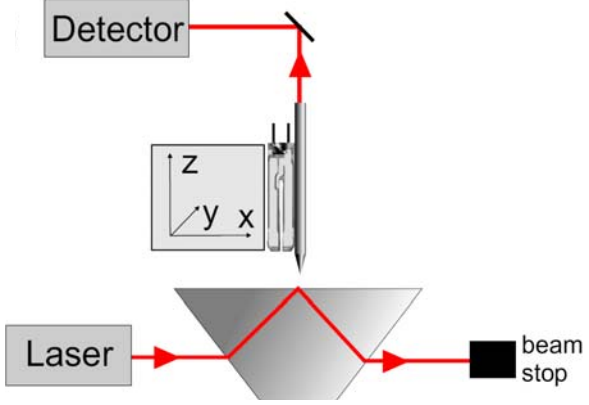






 **STOM/PSTM**


Scanning tunneling optical microscope
Photon scanning tunneling microscope



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 **Evanescent fields**

with increasing θ_1 the argument of the square root in the expression of k_{z2} gets smaller and smaller and eventually becomes negative. The critical angle θ_c can be defined by the condition

$$[1 - \tilde{n}^2 \sin^2 \theta_1] = 0 \rightarrow \theta_c = \arcsin [1/\tilde{n}]$$

$$\varepsilon_2 = 1, \varepsilon_1 = 2.25 \rightarrow \theta_c = 41.8^\circ$$

For $\theta_1 > \theta_c$, k_{z2} becomes imaginary

$$\mathbf{E}_2 = \begin{bmatrix} -i\mathbf{E}_1^{(p)} t^p(\theta_1) \sqrt{\tilde{n}^2 \sin^2 \theta_1 - 1} \\ \mathbf{E}_1^{(s)} t^p(\theta_1) \\ \mathbf{E}_1^{(p)} t^p(\theta_1) \tilde{n} \sin \theta_1 \end{bmatrix} e^{i \sin \theta_1 k_1 x} e^{-\gamma z}$$

$$\gamma = k_1 \sqrt{\tilde{n}^2 \sin^2 \theta_1 - 1}$$

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Evanescent fields

(a) $\theta_i = 45^\circ$ and $\varepsilon_2 = 1, \varepsilon_1 = 2.25$
 $\rightarrow \gamma = 2.22/\lambda$

(b)

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STOM/PSTM

Frustrated total internal reflection

intensity [a.u.]

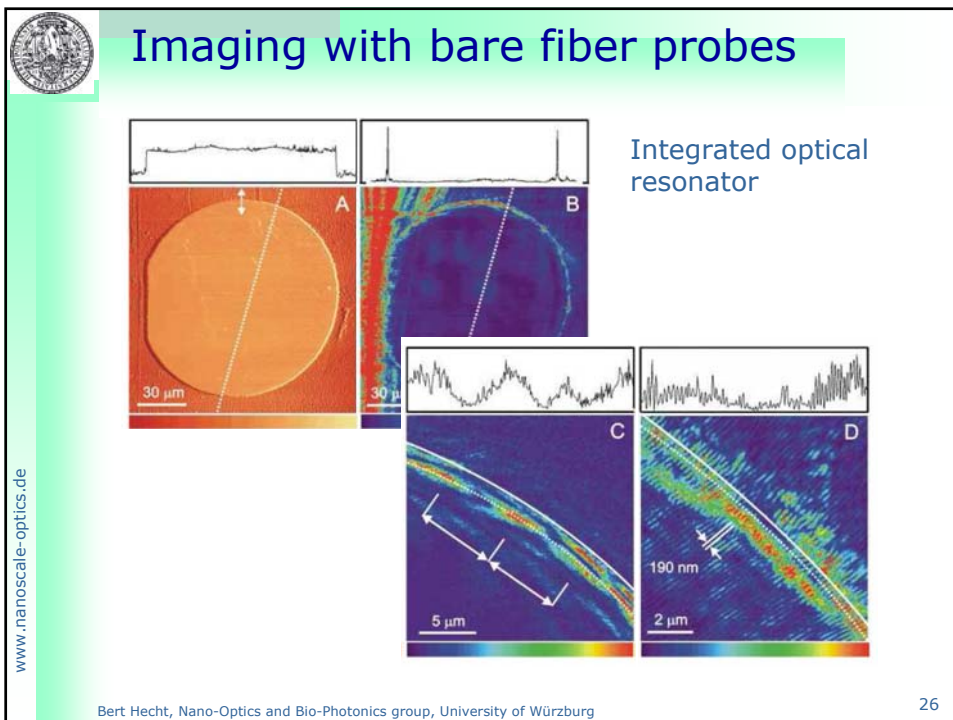
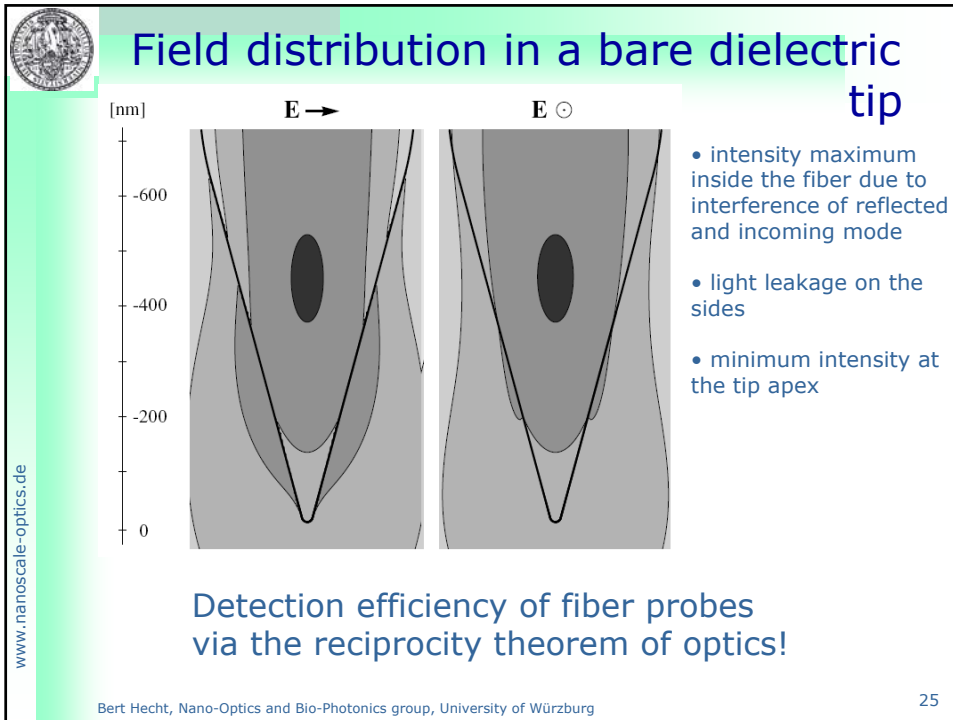
gapwidth [nm]

Bare fiber tips: **+ little disturbance of the field distribution around structures to be studied**
- confinement is limited

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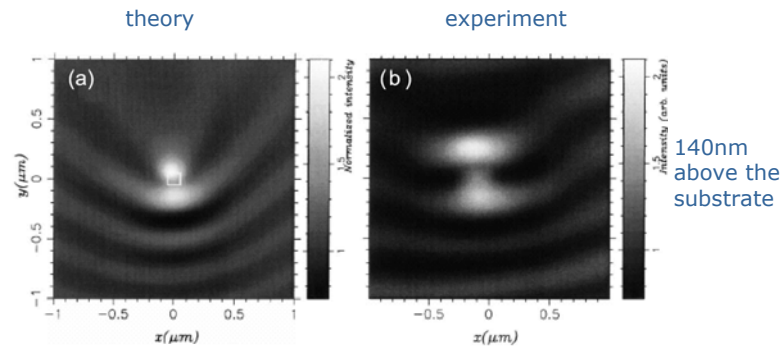
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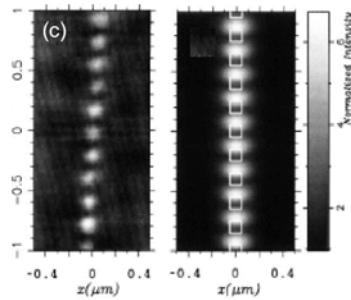
Imaging with bare fiber probes




$100 \times 100 \times 40 \text{ nm}^3$ gold particle
fabricated by electron beam lithography on a
transparent ITO substrate

Near-field distribution is measured

→ Not the shape of the structure!






An electromagnetic field is characterized by **amplitude**, **phase**, and **polarisation state**

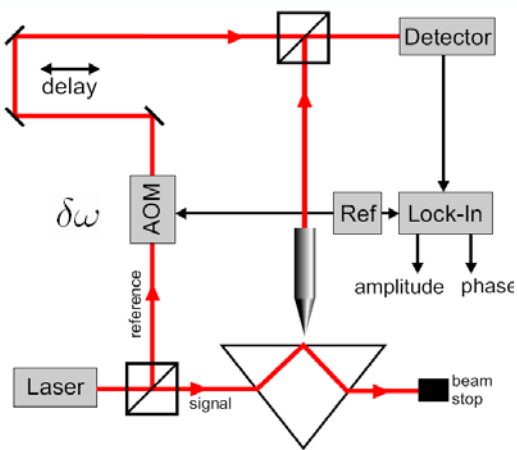
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Amplitude and phase measurement




$\mathbf{E}_S(x, y) = \mathbf{A}_S(x, y) \exp[i(\omega_0 t + \phi_S(x, y) + \beta_S)]$ tip field
 $\mathbf{E}_R = \mathbf{A}_R \exp[i(\omega_0 t + \delta\omega t + \beta_R)]$ reference field

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Tip field and reference field interfere on the detector:


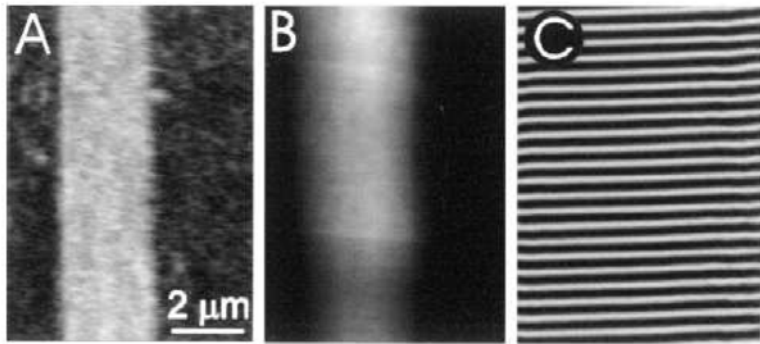
$$I = |\mathbf{A}_S(x, y)|^2 + |\mathbf{A}_R|^2 + 2\mathbf{A}_R \mathbf{A}_S(x, y) \cos [\delta\omega t + \phi_S(x, y) + \beta_S - \beta_R]$$

Demodulation at $\delta\omega$ yields both amplitude $2\mathbf{A}_R \mathbf{A}_S(x, y)$ and phase $\phi_S(x, y)$ of the near-field distribution.

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Field distribution above a Si_3N_4 channel waveguide. Linearly polarized light has been coupled into the channel waveguide to excite only the TM_{00} mode. (a) Topography recorded with shear-force feedback. (b) Amplitude of the field distribution. (c) Cosine of the measured phase distribution.

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Modulation techniques

Linearly polarized light has been coupled in the channel waveguide to excite the lowest TE and TM modes simultaneously. (a) Measured amplitude of the optical field inside the waveguide. A clear beating pattern is observed. (b) Measured phase distribution of the optical field. The cosine of the phase is shown. Several phase singularities are apparent.

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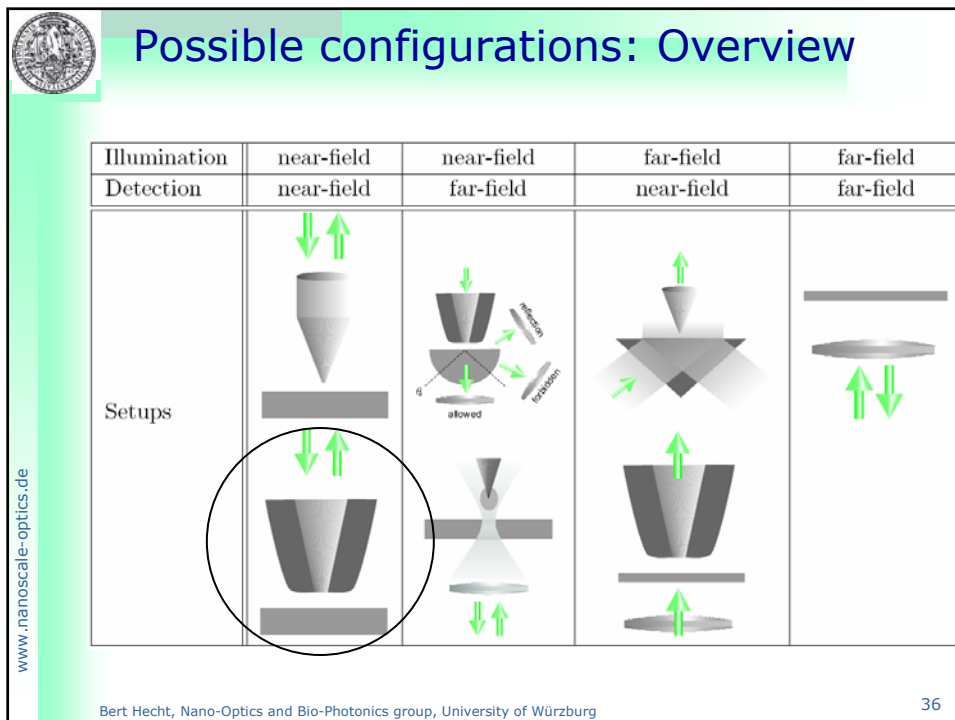
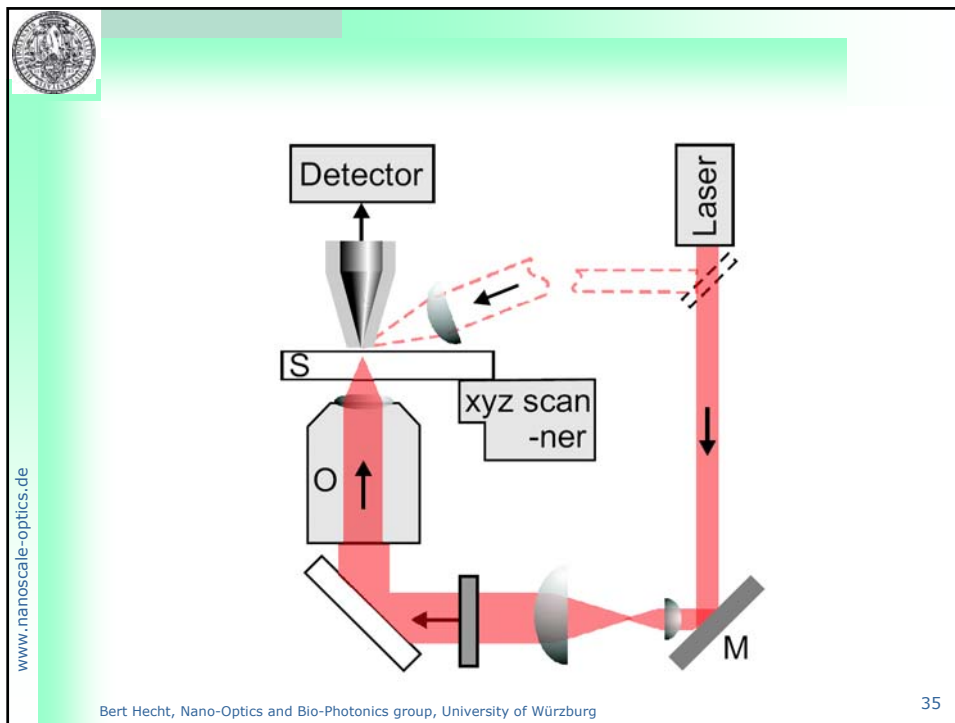
Possible configurations: Overview

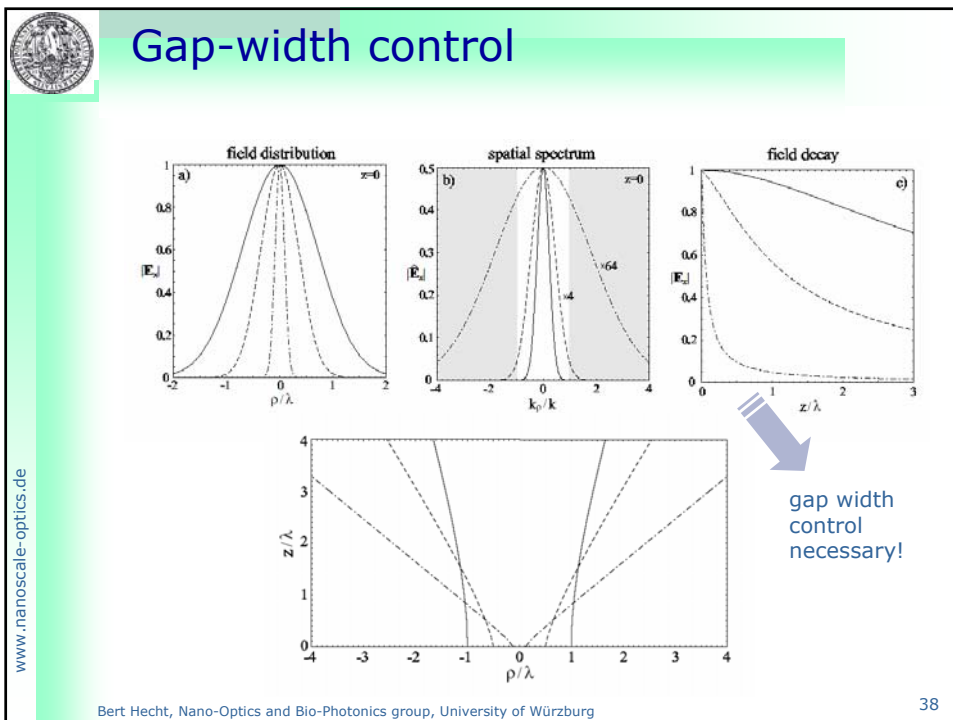
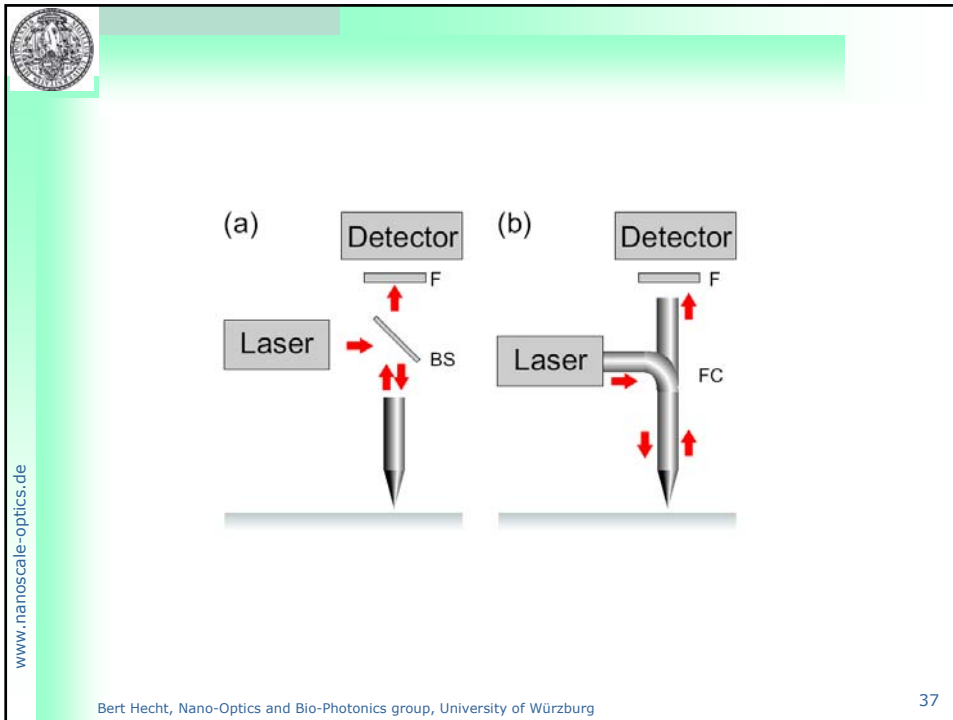
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Gapwidth control

$$I(\omega) = \text{setpoint} \left[\frac{G(\omega) A(\omega) P(\omega)}{1 + G(\omega) A(\omega) P(\omega)} \right]$$

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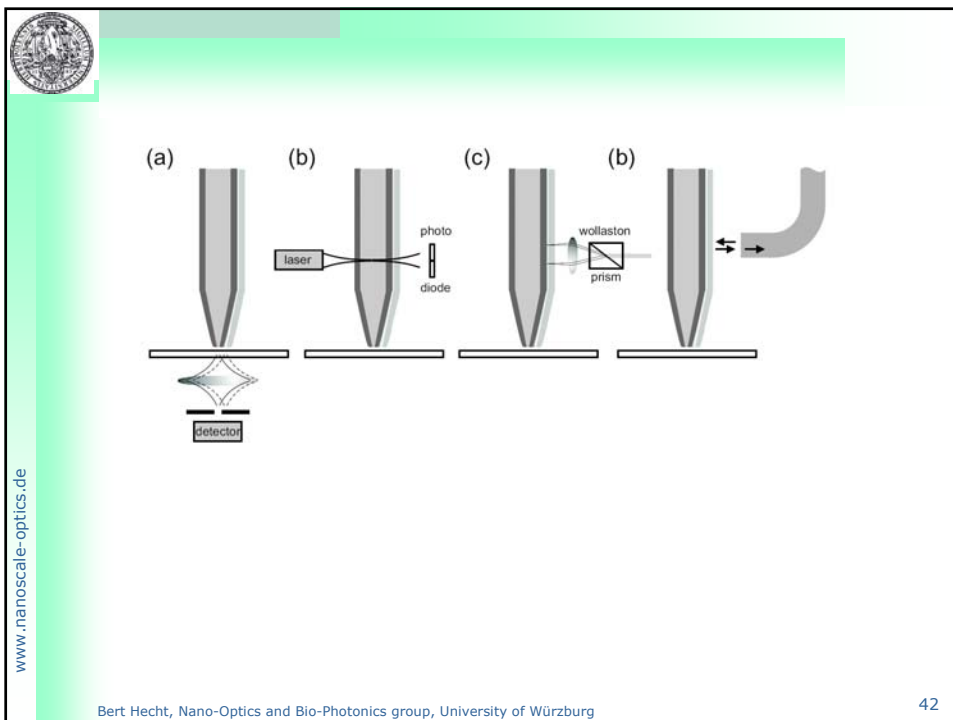
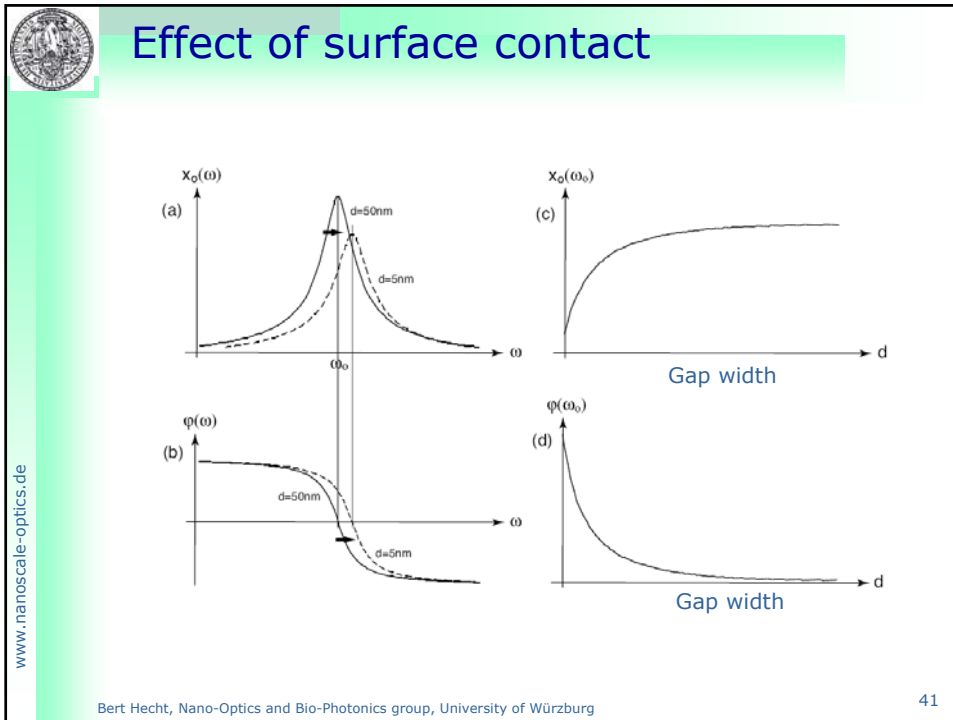
$$\omega_o = 1.76 \sqrt{\frac{E}{\rho}} \frac{R}{L^2}$$


For the example of an optical fiber with radius $R=125\mu\text{m}$ and length $L=3\text{mm}$ we obtain $f_o = \omega_o/(2\pi) \sim 20 \text{ kHz}$.

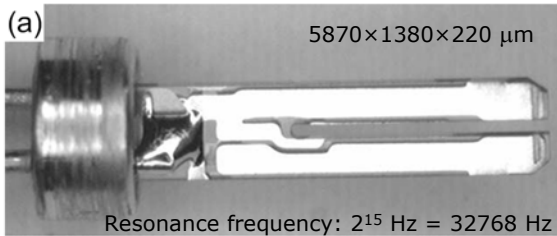
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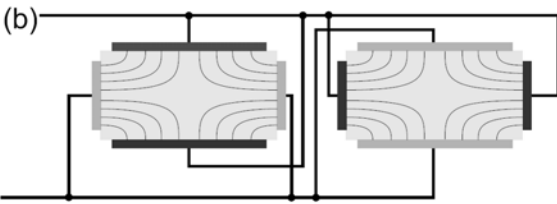
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 **Tuning fork sensor**


(a)  5870×1380×220 μm
Resonance frequency: 2^{15} Hz = 32768 Hz

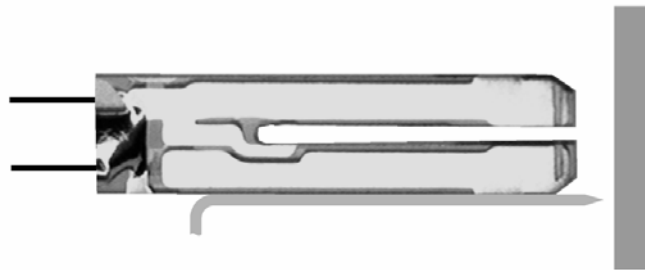
(b) 

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 **Tuning fork sensor**



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Effective harmonic oscillator model

friction-force elastic force

$$m \ddot{x}(d, t) + m \gamma(d) \dot{x}(d, t) + m \omega_o^2(d) x(d, t) = F e^{-i\omega t}$$

Damping constant \rightarrow $m \gamma(d)$
 Resonance frequency $f_o = \omega_o / 2\pi$ \rightarrow $m \omega_o^2(d)$
 Driving force \rightarrow $F e^{-i\omega t}$

d ... gap width

Steady-state solution:


$$x(t) = \frac{(F/m)}{\omega_o^2 - \omega^2 - i\gamma\omega} e^{-i\omega t} \quad Q = \frac{f_o}{\Delta f} = \frac{\omega_o}{\gamma\sqrt{3}}$$

Lorentzian line shape
with quality factor:

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steady-state solution for $\omega = \omega_o$:

$$\gamma(d) = \gamma_{stat} + \gamma_{int}(d) = \frac{(F/m)}{\omega_o(d) x_o(d)}$$


$$\gamma_{int}(d \rightarrow \infty) = 0 \rightarrow$$

$$\gamma_{int}(d) = \gamma_{stat} \left[\frac{\omega_o(\infty) x_o(\infty)}{\omega_o(d) x_o(d)} - 1 \right]$$

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$$F_{int}^{friction}(d) = m \gamma_{int}(d) \omega_o(d) x_o(d) =$$

$$\left[1 - \frac{\omega_o(d) x_o(d)}{\omega_o(\infty) x_o(\infty)} \right] \frac{k_{stat} x_o(\infty)}{\sqrt{3} Q(\infty)}$$

using $m = k_{stat} / \omega_o^2(\infty)$
and $Q = \frac{f_o}{\Delta f} = \frac{\omega_o}{\gamma \sqrt{3}}$

↑
~1


$$F_{int}^{friction}(d) = \left[1 - \frac{V(d)}{V(\infty)} \right] \frac{k_{stat}}{\sqrt{3} Q(\infty)} x_o(\infty)$$

using that the displacement x_o is directly proportional to the voltage V

→ Calculation of the friction force

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$$F_{int}^{friction}(d) = \left[1 - \frac{V(d)}{V(\infty)} \right] \frac{k_{stat}}{\sqrt{3} Q(\infty)} x_o(\infty)$$

↑
~0.1

$k_{stat} = 40 \text{ kN/m}$
 $x_o(\infty) = 10 \text{ pm}$
 $Q(\infty) \approx 1200$

→ $F_{int}^{friction} \approx 20 \text{ pN}$

thermal noise:

$$\frac{1}{2} k_{stat} x_{rms}^2 = \frac{1}{2} k_B T \quad \rightarrow \quad x_{rms} = 0.32 \text{ pm}$$

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Elastic force

$$m = \frac{k_{stat} + k_{int}(d)}{\omega_o^2(d)} = \frac{k_{stat}}{\omega_o^2(\infty)} \rightarrow$$

$$k_{int}(d) = k_{stat} \left[\frac{\omega_o^2(d)}{\omega_o^2(\infty)} - 1 \right]$$

$$F_{int}^{elastic}(d) = k_{int}(d) x_o(d) = \left[\frac{\omega_o^2(d)}{\omega_o^2(\infty)} - 1 \right] k_{stat} x_o(d)$$

frequency shift 5 Hz

$x_o(d) = 9$ pm (reduction by 10%)

$k_{stat} = 40$ kN/m

$$\rightarrow F_{int}^{elastic} \approx 110 \text{ pN}$$