Nano-Optics

Bert Hecht

Nano-Optics and Bio-Photonics Group
Lehrstuhl Experimentelle Physik V

Universität Würzburg
Am Hubland
97074 Würzburg

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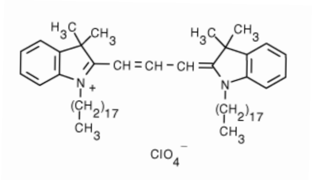
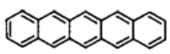
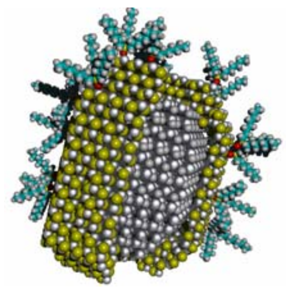
Single emitters

Organic dye molecules





Semiconductor nanocrystal quantum dots



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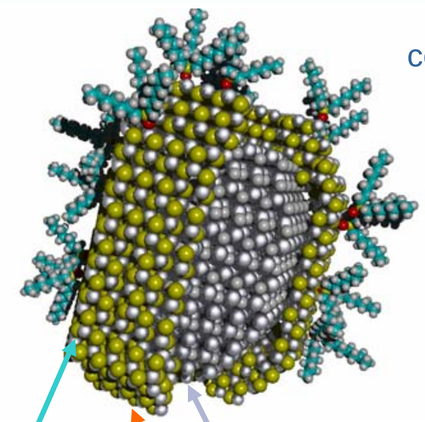
Colloidal Semiconductor Nanocrystals



complex architecture

Core: opto-electronically active part

Shell: protective, complementary part

Organics: protective, functional part → biological applications

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Slides courtesy H.-J. Eisler

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Photoluminescence Polarization

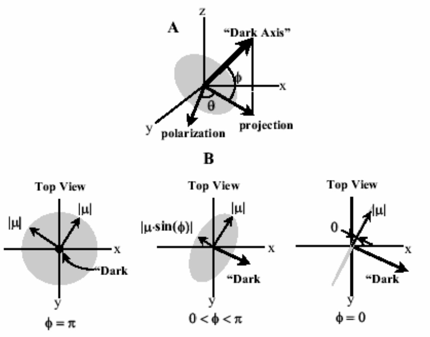
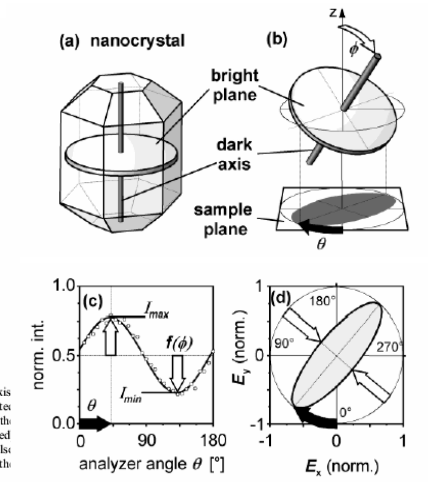


Figure 5.5: "Dark axis" coordinate system with light propagation along the z-axis (A) The shaded disk represents the planar, degenerate transition dipole oriented normal to the "dark axis". (B) Top views depict the projection of the dipole onto the sample (x-y) plane at three different values of ϕ. This projection (squared corresponds to the polarization modulation measured by the detection system. Also included are the magnitudes of two orthogonal dipole projections, as well as the projection of the "dark axis" onto the sample plane at each angle.



Empedocles, Neuhauser, Bawendi
Nature 1999, 399:126-130.

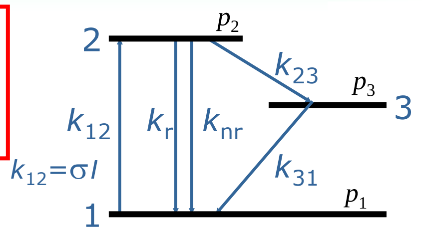
F. Koberling *et al.*, *J. Phys. Chem. B* **2003**, *107*, 7463 (2003)

Detecting single emitters?

Signal size estimation:

Steady-state emission rate

$$R = k_r p_2$$



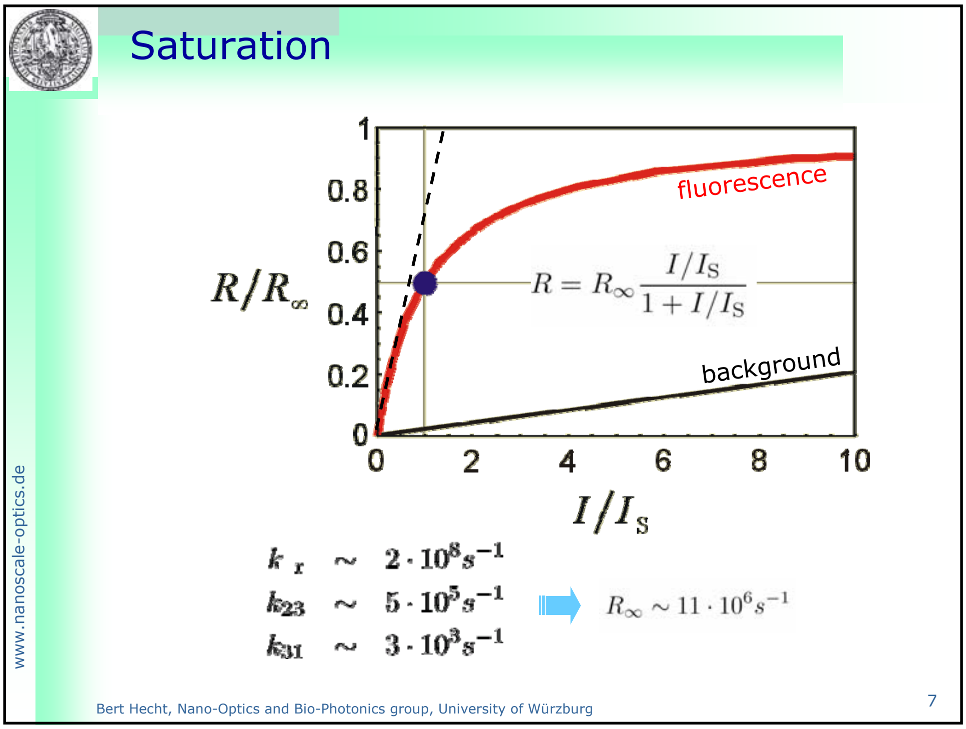
simplified Jablonski diagram

$$R = R_\infty \frac{I/I_S}{1 + I/I_S}$$

$$k_{12} = \sigma I$$

$$R_\infty = \frac{k_{31}\; k_r}{k_{23} + k_{31}}$$

$$I_S = \frac{(k_r + k_{nr} + k_{23})k_{31}}{\sigma(k_{23} + k_{31})}$$


Saturation
fluorescence
background
R/R_∞
I/I_S
$R = R_\infty \frac{I/I_S}{1+I/I_S}$
0 0.2 0.4 0.6 0.8 1
0 2 4 6 8 10
$k_r \sim 2\cdot10^8 s^{-1}$
$k_{23} \sim 5\cdot10^5 s^{-1}$
$k_{31} \sim 3\cdot10^3 s^{-1}$
$R_\infty \sim 11\cdot10^6 s^{-1}$
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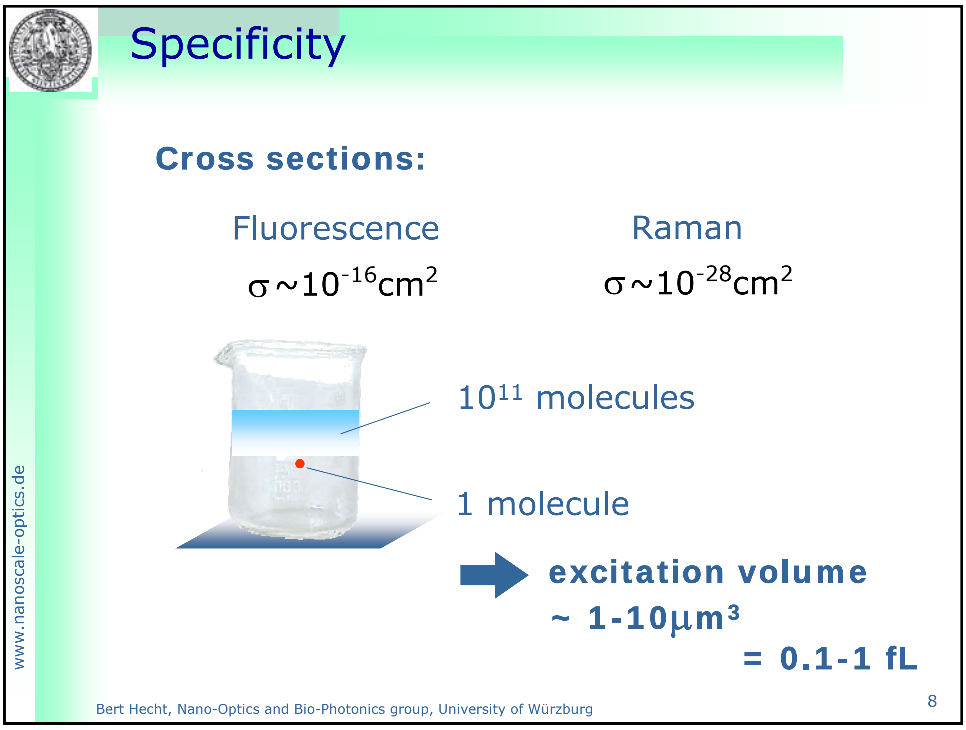
Specificity
Cross sections:
Fluorescence
$\sigma \sim 10^{-16} cm^2$
Raman
$\sigma \sim 10^{-28} cm^2$
10^{11} molecules
1 molecule
excitation volume
~ 1-10μm³
= 0.1-1 fL
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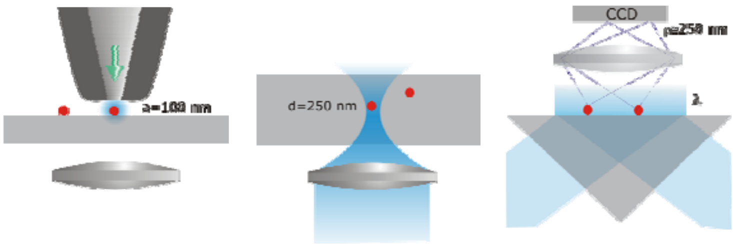
Microscopy techniques

- control of background signals
- isolate single molecules by spatial (and spectral) selection & dilution



$V = \pi a^2/4 \cdot a$
$= 8 \cdot 10^{-4} \mu m^3$

$V = \pi d^2/4 \cdot 2.5d$
$= 3 \cdot 10^{-2} \mu m^3$

$V = \pi \rho^2/4 \cdot \lambda$
$= 2.5 \cdot 10^{-2} \mu m^3$

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Quantum properties of single emitters

Single emitters emit single photons at a time

Photon Statistics

analysis of the time structure (fluctuations) in the stream of emitted photons

→ a way to identify possible "nonclassical" or quantum behavior of light

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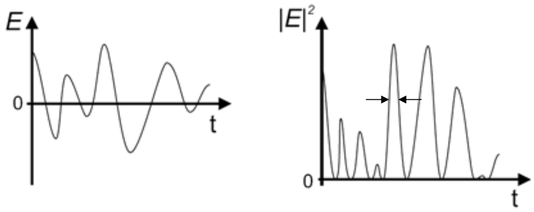
Tool: Second order autocorr. function



$$g^{(2)}(\tau) = \frac{\langle I(t) I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$

$$\langle \ldots \rangle = \frac{1}{T}\int \ldots dt$$ for large but finite T

Literature: R. Loudon, The quantum theory of light, Oxford University Press

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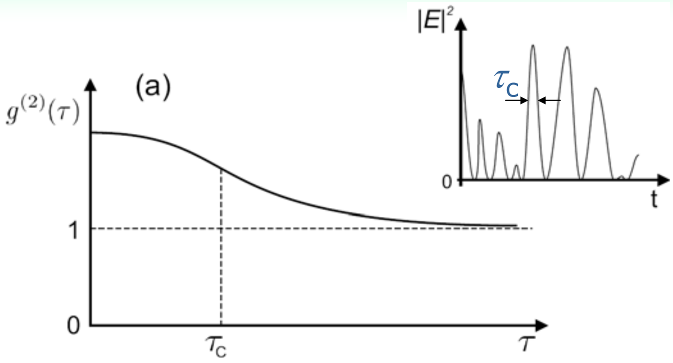
Properties of $g^2(\tau)$



If the intensity is a classical continuous function of time the following relations must be fulfilled:

$$g^{(2)}(0) \geq 1$$
$$g^{(2)}(\tau) \leq g^{(2)}(0)$$

Literature: R. Loudon, The quantum theory of light, Oxford University Press

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Single quantum system

consider

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I\ \ (t)\rangle^2} \quad \text{for} \quad t = 0$$

For t = 0 we prepare the emitter in the ground state: $p_2(0) = 0$

The probability for any photon to be **detected** at time τ provided a photon was emitted at $\tau = 0$ is then given by $\eta k_{\mathrm{r}} p_2(\tau)$

$$g^{(2)}(\tau) = \frac{\eta k_{\mathrm{r}} p_2(\tau)}{\eta k_{\mathrm{r}} p_2(\infty)} = \frac{p_2(\tau)}{p_2(\infty)}$$

$\eta k_{\mathrm{r}} p_2(\infty)$ is the steady-state count-rate that is measured after a sufficiently long time

we need the time-dependent solution of

$$\begin{aligned}
\dot{p}_1 &= -k_{12}p_1 + (k_{\mathrm{r}} + k_{\mathrm{nr}})p_2 + k_{31}p_3 \\
\dot{p}_2 &= k_{12}p_1 - (k_{\mathrm{r}} + k_{\mathrm{nr}} + k_{23})p_2 \\
\dot{p}_3 &= k_{23}p_2 - k_{31}p_3
\end{aligned}$$

$$\begin{aligned}
\dot{p}_1 &= -(k_{12} + k_{31})p_1 + (k_{\mathrm{r}} + k_{\mathrm{nr}} - k_{31})p_2 + k_{31} \\
\dot{p}_2 &= k_{12}p_1 - (k_{\mathrm{r}} + k_{\mathrm{nr}} + k_{23})p_2\ .
\end{aligned}$$

can be written in matrix form

$$\dot{\mathbf{p}}(\tau) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mathbf{p}(\tau) + \begin{bmatrix} f \\ 0 \end{bmatrix}$$

Laplace transformation:

$$s\mathbf{p}(s) - \mathbf{p}(0) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mathbf{p}(s) + \frac{1}{s} \begin{bmatrix} f \\ 0 \end{bmatrix}$$

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Laplace transformation

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt.$$ with $s = \sigma + \mathrm{i}\omega; \quad \sigma > 0; \quad t \geq 0$

$F(s)$ is called the Laplace-transform of $f(t)$

Inverse transformation:

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{st} F(s)\, ds \qquad \gamma > \max_k \mathrm{Re}(z_k),$$

where the z_k are the singularities of F

Calculation of the integrals via complex analysis (Residuensatz, usw.)

Most of the times transformation possible using tables of transformations

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Transformation table

f	$\mathcal{L}_t[f(t)](s)$				
	$\frac{1}{s}$	$s>0$			
t	$\frac{1}{s^2}$	$s>0$	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$	$s>a$
t^n	$\frac{n!}{s^{n+1}}$	$n\in\mathbb{Z}>0$			
t^a	$\frac{\Gamma(a+1)}{s^{a+1}}$	$a>0$	$\delta(t-c)$	e^{-cs}	$c>0$
e^{at}	$\frac{1}{s-a}$	$s>a$	$H_c(t)$	$\frac{e^{-cs}}{s}$	$s>0$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$s>0$	$J_0(t)$	$\frac{1}{\sqrt{s^2+1}}$	
$\sin(at)$	$\frac{a}{s^2+a^2}$	$s>0$			
$\cosh(at)$	$\frac{s}{s^2-a^2}$	$s>\lvert a\rvert$	$J_n(at)$	$\frac{\left(\sqrt{s^2+a^2}-s\right)^n}{a^n\sqrt{s^2+a^2}}$	$s>0, n>-1$
$\sinh(at)$	$\frac{a}{s^2-a^2}$	$s>\lvert a\rvert$			
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	$s>a$			

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Properties of the Laplace transform

http://de.wikipedia.org/wiki/Laplace-Transformation

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Example

Solve the differential equation,

$$5\frac{dy}{dt}+4y=2, \quad y(0)=1$$

using Laplace transform.

Solution) Take Laplace transform both sides

$$5\{sY(s)-y(0)\}+4Y(s)=\frac{2}{s}$$

$$Y(s)=\frac{5s+2}{s(5s+4)}$$

$$y(t)=L^{-1}\left\{\frac{5s+2}{s(5s+4)}\right\}=0.5\{1+\exp(-0.8t)\}$$

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$$\mathbf{p}(s) = \left[s \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right]^{-1} \begin{pmatrix} f/s + 1 \\ 0 + 0 \end{pmatrix}$$

using $p_1(0)=1$

back transformation using the Heaviside expansion theorem:

$$p_2(\tau) = A_1 e^{s_1 \tau} + A_2 e^{s_2 \tau} + A_3$$

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$$s_1 = \frac{1}{2}\left(a + d - \sqrt{(a-d)^2 + 4bc}\right)$$

$$s_2 = \frac{1}{2}\left(a + d + \sqrt{(a-d)^2 + 4bc}\right)$$

$$A_1 = +c\frac{1 + f/s_1}{s_1 - s_2}, \quad A_2 = -c\frac{1 + f/s_2}{s_1 - s_2}, \quad A_3 = \frac{cf}{s_1 s_2}$$

using $p_2(\infty) = A_3$

and $-A_1/A_3 = (1 + A_2/A_3)$

$$\Rightarrow \quad g^2(\tau) = -(1 + \frac{A_2}{A_3})e^{s_1\tau} + \frac{A_2}{A_3}e^{s_2\tau} + 1$$

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$$k_{21} \geq k_{12} \gg k_{23} \geq k_{31}$$

$$\begin{aligned} s_1 &\simeq -(k_{12} + k_{21}) \\ s_2 &\simeq -\left(k_{31} + \frac{k_{12}k_{23}}{k_{12} + k_{21}}\right) \\ \frac{A_2}{A_3} &\simeq \frac{k_{12}k_{23}}{k_{31}(k_{12} + k_{21})} \end{aligned}$$

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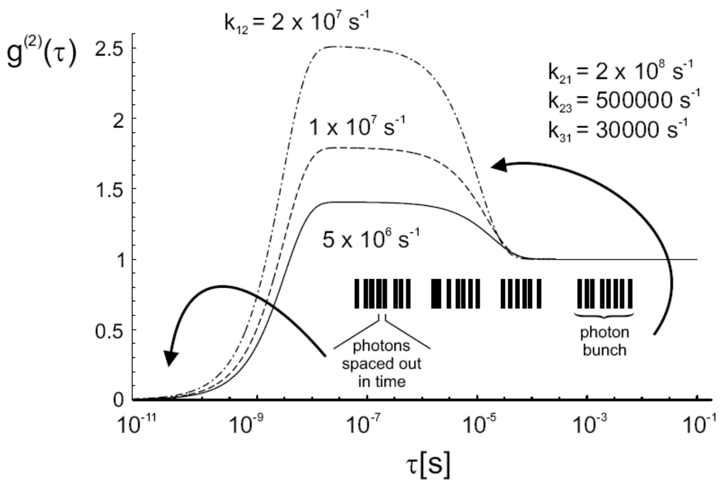

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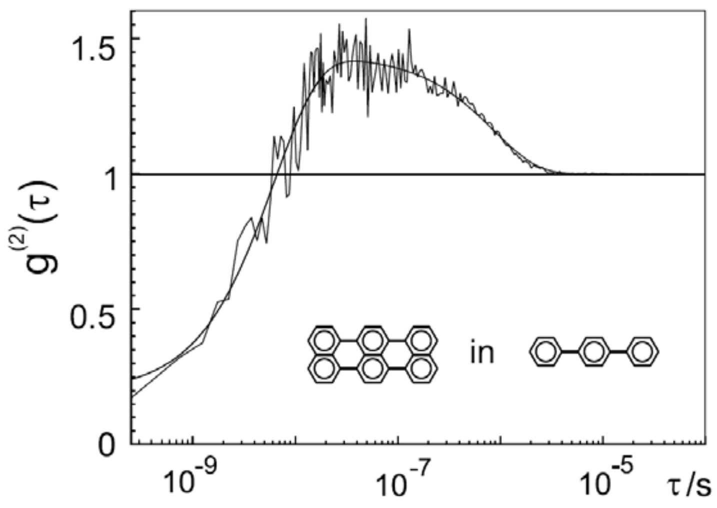
Experiment



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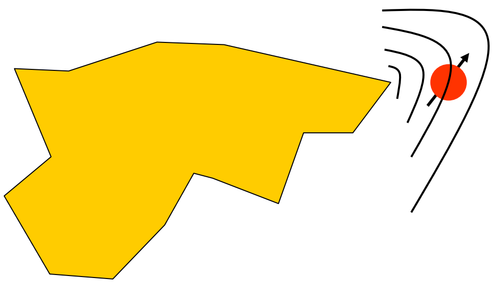
Electrodynamics of single emitters

- although single emitters emit a quantum field there interaction with matter can in most cases be decribed by classical electrodynamics!



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Radiating electric dipole

$$\mathbf{j}(\mathbf{r},t) = \frac{d}{dt}\boldsymbol{\mu}(t)\,\delta[\mathbf{r}-\mathbf{r}_o]$$

with

$$\boldsymbol{\mu}(t) = \sum_n q_n[\mathbf{r}_n(t)-\mathbf{r}_o]$$

harmonic time-dependence:

$$\mathbf{j}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{j}(\mathbf{r})\exp(-i\omega t)\}$$
$$\boldsymbol{\mu}(t) = \mathrm{Re}\{\boldsymbol{\mu}\exp(-i\omega t)\}$$

$$\Rightarrow \quad \mathbf{j}(\mathbf{r}) = -i\omega\boldsymbol{\mu}\,\delta[\mathbf{r}-\mathbf{r}_o]$$

Electric dipole in homogeneous space

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_o + i\omega\mu\mu_o\int_V \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')\,\mathbf{j}(\mathbf{r}')\,dV'$$

als formale Lösung der Helmholtzgleichung

using $\mathbf{j}(\mathbf{r}) = -i\omega\boldsymbol{\mu}\,\delta[\mathbf{r}-\mathbf{r}_o]$

$$\Rightarrow \quad \mathbf{E}(\mathbf{r}) = \omega^2\mu\mu_o\,\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_o)\,\boldsymbol{\mu}$$

with $\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_o) = \left[\overleftrightarrow{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\right]G(\mathbf{r},\mathbf{r}_o)$

where $G(\mathbf{r},\mathbf{r}_o) = \frac{\exp(ik|\mathbf{r}-\mathbf{r}_o|)}{4\pi|\mathbf{r}-\mathbf{r}_o|}$

In a Cartesian coordinate system:

$$\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_o) = \frac{\exp(ikR)}{4\pi R}\left[\left(1+\frac{ikR-1}{k^2R^2}\right)\overleftrightarrow{\mathbf{I}} + \frac{3-3ikR-k^2R^2}{k^2R^2}\frac{\mathbf{RR}}{R^2}\right]$$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}_o$$

$\mathbf{RR}$ denotes the outer product of $\mathbf{R}$ with itself.

$$\overleftrightarrow{\mathbf{G}} = \overleftrightarrow{\mathbf{G}}_{NF} + \overleftrightarrow{\mathbf{G}}_{IF} + \overleftrightarrow{\mathbf{G}}_{FF}$$

$$(kR)^{-3} \qquad (kR)^{-2} \qquad (kR)^{-1}$$

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$$\overleftrightarrow{\mathbf{G}}_{NF} = \frac{\exp(ikR)}{4\pi R}\frac{1}{k^2R^2}\left[-\overleftrightarrow{\mathbf{I}} + 3\mathbf{RR}/R^2\right]$$

$$\overleftrightarrow{\mathbf{G}}_{IF} = \frac{\exp(ikR)}{4\pi R}\frac{i}{kR}\left[\overleftrightarrow{\mathbf{I}} - 3\mathbf{RR}/R^2\right]$$

$$\overleftrightarrow{\mathbf{G}}_{FF} = \frac{\exp(ikR)}{4\pi R}\left[\overleftrightarrow{\mathbf{I}} - \mathbf{RR}/R^2\right]$$

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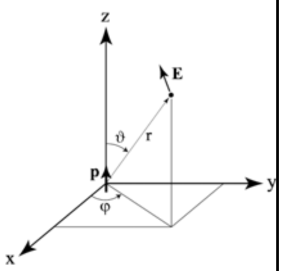
in homogeneous space it is sufficient to consider $\boldsymbol{\mu} = |\boldsymbol{\mu}|\,\mathbf{n}_z$

and use spherical coordinates: $\mathbf{r} = (r, \vartheta, \varphi)$



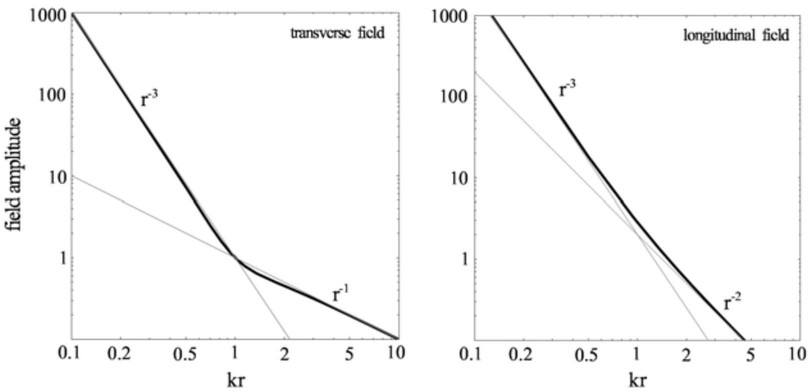
➡ $\mathbf{E} = (E_r, E_\vartheta, E_\varphi)$

$$E_r = \frac{|\boldsymbol{\mu}|\cos\vartheta}{4\pi\varepsilon_o\varepsilon}\,\frac{\exp(ikr)}{r}\,k^2\left[\frac{2}{k^2r^2} - \frac{2i}{kr}\right],$$

$$E_\vartheta = \frac{|\boldsymbol{\mu}|\sin\vartheta}{4\pi\varepsilon_o\varepsilon}\,\frac{\exp(ikr)}{r}\,k^2\left[\frac{1}{k^2r^2} - \frac{i}{kr} - 1\right]$$

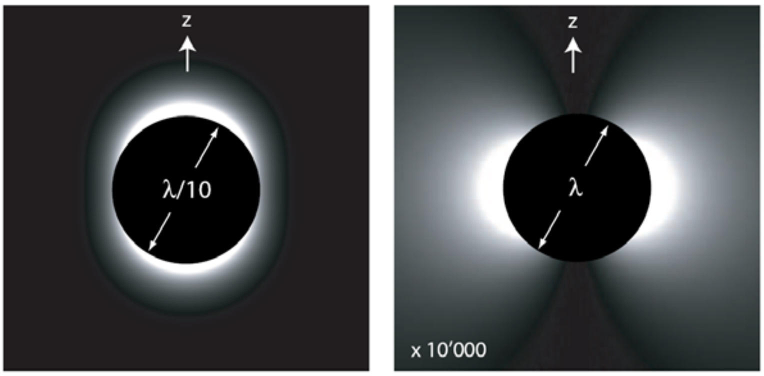
$$H_\varphi = \frac{|\boldsymbol{\mu}|\sin\vartheta}{4\pi\varepsilon_o\varepsilon}\,\frac{\exp(ikr)}{r}\,k^2\left[-\frac{i}{kr} - 1\right]\sqrt{\frac{\varepsilon_o\varepsilon}{\mu_o\mu}}$$

E_r has no farfield term → field in the farfield is purely transverse
magnetic field has no terms in $(kr)^{-3}$ → the nearfield is dominated by the electric field



While both the transverse and the longitudinal field contribute to the near field, only the transverse field survives in the far field.

Near field vs. far field



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Dipole radiation

Only the far field contributes to net energy transport (in free space)

$$\mathbf{S}(t) = \mathbf{E}(t) \times \mathbf{H}(t)$$

$$\boldsymbol{\mu}(t) = \mathrm{Re}\{\boldsymbol{\mu} \exp(-i\omega t)\}$$

$$P(t) = \int_{\partial V} \mathbf{S} \cdot \mathbf{n} \, da = \frac{1}{4\pi\varepsilon_o\varepsilon} \frac{2\,n^3}{3\,c^3} \left[\frac{d^2\,|\boldsymbol{\mu}(t)|}{dt^2}\right]^2$$

$$\bar{P} = \frac{|\boldsymbol{\mu}|^2}{4\pi\varepsilon_o\varepsilon} \frac{n^3\omega^4}{3\,c^3}$$

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Poyntings theorem

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}, \quad (1)$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} + \mathbf{j}(\mathbf{r},t), \quad (2)$$

$\mathbf{H} \cdot (1)$ - $\mathbf{E} \cdot (2)$

$$\mathbf{H}\cdot(\nabla\times\mathbf{E}) - \mathbf{E}\cdot(\nabla\times\mathbf{H}) = -\mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t} - \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t} - \mathbf{j}\cdot\mathbf{E}$$

$$= \nabla\cdot(\mathbf{E}\times\mathbf{H})$$

integrating both
sides over space and applying Gauss' theorem

$$\int_{\partial V} (\mathbf{E}\times\mathbf{H})\cdot\mathbf{n}\, da = -\int_V \left[\mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t} + \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}\cdot\mathbf{E}\right] dV$$

$$\mathbf{D}(\mathbf{r},t) = \varepsilon_o \mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t),$$

$$\mathbf{H}(\mathbf{r},t) = \mu_o^{-1}\mathbf{B}(\mathbf{r},t) - \mathbf{M}(\mathbf{r},t)$$

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Energy conservation

Flow of e.m. energy in or out of V

time rate of change of electromagnetic energy inside V

$$\int_{\partial V} (\mathbf{E}\times\mathbf{H})\cdot\mathbf{n}\, da + \frac{1}{2}\frac{\partial}{\partial t}\int_V \left[\mathbf{D}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{H}\right] dV =$$

$$-\int_V \mathbf{j}\cdot\mathbf{E}\, dV$$

for linear media

Energy dissipation in V
= Ohmic losses

$$\mathbf{S} = (\mathbf{E}\times\mathbf{H})$$

energy flux density, Poynting vector

time- averaged Poynting vector

$$\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\}$$

$$W = \frac{1}{2}\left[\mathbf{D}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{H}\right]$$

density of electromagnetic energy

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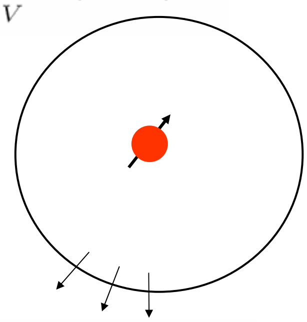
the radiated power of any current distribution with a harmonic time dependence in a linear medium has to be identical to the rate of energy dissipation:

$$\frac{dW}{dt} = -\frac{1}{2}\int_V \mathrm{Re}\{\mathbf{j}^* \cdot \mathbf{E}\}\, dV$$

flow of energy out of V



$$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$$

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using $\mathbf{j}(\mathbf{r}) = -i\omega\boldsymbol{\mu}\,\delta[\mathbf{r}-\mathbf{r}_o]$

$$\frac{dW}{dt} = \frac{\omega}{2}\,\mathrm{Im}\{\boldsymbol{\mu}^* \cdot \mathbf{E}(\mathbf{r}_o)\}$$

evaluated at the dipole's origin $\boldsymbol{r}_o$

in terms of the Green's function $\mathbf{E}(\mathbf{r}) = \omega^2\mu\mu_o\,\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_o)\,\boldsymbol{\mu}$

$$\frac{dW}{dt} = \frac{\omega^3\,|\boldsymbol{\mu}|^2}{2\,c^2\varepsilon_o\varepsilon}\left[\mathbf{n}_\mu \cdot \mathrm{Im}\left\{\overleftrightarrow{\mathbf{G}}(\mathbf{r}_o,\mathbf{r}_o;\omega)\right\}\cdot \mathbf{n}_\mu\right]$$

unit vector in direction of the dipole moment

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At first sight it seems not possible to evaluate

$$\frac{dW}{dt} = \frac{\omega^3 |\boldsymbol{\mu}|^2}{2c^2\varepsilon_o\varepsilon}\left[\mathbf{n}_\mu \cdot \mathrm{Im}\left\{\overleftrightarrow{\mathbf{G}}(\mathbf{r}_o, \mathbf{r}_o; \omega)\right\} \cdot \mathbf{n}_\mu\right]$$

since $\exp(ikR)/R$ appears to be infinite at $\boldsymbol{r} = \boldsymbol{r}_o$.

This is however not the case:

Due to the dot product between $\boldsymbol{\mu}$ and $\boldsymbol{E}$ we need only to evaluate the component of $\boldsymbol{E}$ in direction of $\boldsymbol{\mu}$

Choose $\boldsymbol{\mu} = |\boldsymbol{\mu}|\,\mathbf{n}_z$

$$E_z = \frac{|\boldsymbol{\mu}|}{4\pi\varepsilon_o\varepsilon}\frac{e^{ikR}}{R}\left[k^2 \sin^2\vartheta + \frac{1}{R^2}(3\cos^2\vartheta - 1) - \frac{ik}{R}(3\cos^2\vartheta - 1)\right]$$

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Consider the limiting case $R \longrightarrow 0$

$$\exp(ikR) = 1 + ikR + (1/2)(ikR)^2 + (1/6)(ikR)^3 + ..$$

$$\frac{dW}{dt} = \lim_{R\to 0} \frac{\omega}{2} |\boldsymbol{\mu}| \,\mathrm{Im}\{E_z\} =$$

$$\frac{\omega\,|\boldsymbol{\mu}|^2}{8\pi\varepsilon_o\varepsilon} \lim_{R\to 0}\left\{\frac{2}{3}k^3 + R^2(..) + ..\right\} = \frac{|\boldsymbol{\mu}|^2}{12\pi}\frac{\omega}{\varepsilon_o\varepsilon}k^3$$

correct result despite of the apparent singularity at R=0!

compare to $\bar{P} = \frac{|\boldsymbol{\mu}|^2}{4\pi\varepsilon_o\varepsilon}\frac{n^3\omega^4}{3\,c^3}$

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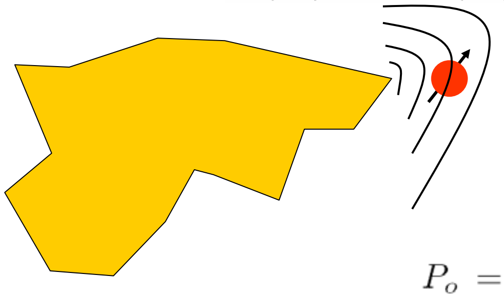
Dipole in inhomogeneous space

$$\mathbf{E}(\mathbf{r}_o) = \mathbf{E}_o(\mathbf{r}_o) + \mathbf{E}_s(\mathbf{r}_o)$$



$$P_o = \frac{|\boldsymbol{\mu}|^2}{12\pi}\frac{\omega}{\varepsilon_o\varepsilon}k^3$$

$P = P_0 + P_S$

$P/P_0 = 1 + P_S/P_0$

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Rate of energy dissipation in inhomogeneous space

$$\frac{P}{P_o} = 1 + \frac{6\pi\varepsilon_o\varepsilon}{|\boldsymbol{\mu}|^2}\frac{1}{k^3}\,\mathrm{Im}\{\boldsymbol{\mu}^* \cdot \mathbf{E}_s(\mathbf{r}_o)\}$$

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Another point of view

undriven harmonically oscillating dipole:

$$\frac{d^2}{dt^2}\boldsymbol{\mu}(t) + \gamma_o \frac{d}{dt}\boldsymbol{\mu}(t) + \omega_o^2 \boldsymbol{\mu}(t) = 0$$

$$\Rightarrow \quad \boldsymbol{\mu}(t) = \mathrm{Re}\left\{\boldsymbol{\mu}_o \, \mathrm{e}^{-i\omega_o\sqrt{1-(\gamma_o^2/4\omega_o^2)}\,t} \, \mathrm{e}^{-\gamma_o t/2}\right\}$$

damped oscillation with slight frequency shift

require that $\gamma_o \ll \omega_o$

then the average energy stored in the oscillator is:

$$\bar{W}(t) = \frac{m}{2q^2}\left[\omega_o^2 \mu^2(t) + \dot{\mu}^2(t)\right] = \frac{m\omega_o^2}{2q^2}\,|\boldsymbol{\mu}_o|^2\,\mathrm{e}^{-\gamma_o t}$$

$\Rightarrow$ lifetime of the oscillator: $\tau_o = 1/\gamma_o$

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Classical decay rate

Energy conservation requires that the decrease in oscillator energy must equal the energy losses

quantum yield

$$\bar{W}(t\!=\!0) - \bar{W}(t) = q_i \int_0^t P_o(t')\,dt'$$

using $\bar{W}(t)$ as just defined and $P_o(t) = \frac{|\boldsymbol{\mu}(t)|^2}{4\pi\varepsilon_o}\frac{\omega_o^4}{3\,c^3}$

we get

$$\gamma_o = q_i \frac{1}{4\pi\varepsilon_o}\frac{2q^2\omega_o^2}{3\,m\,c^3}$$

classical expression for the atomic decay rate

At optical wavelengths a value for the decay rate of ~$2\cdot10^{-8}\,\mathrm{s}^{-1}$ is obtained.

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