Nano-Optics

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Table of Contents

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General remarks

- Light fields are treated in the wave picture (Maxwell's equations)
- Microscopic matter is treated by Quantum Mechanics

→ Semiclassical description

Exceptions: non-classical fields emitted by single emitters!

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Maxwell's equations

Maxwell:

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \quad (1)$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} + \mathbf{j}(\mathbf{r},t) \quad (2)$$

$$\nabla \cdot \mathbf{D}(\mathbf{r},t) = \rho(\mathbf{r},t) \quad (3)$$

$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0 \quad (4)$$

macroscopic Maxwell equations

Conservation of charge:

$$\nabla \cdot \mathbf{j}(\mathbf{r},t) + \frac{\partial \rho(\mathbf{r},t)}{\partial t} = 0$$

From (2) using $\nabla \cdot \nabla \times \mathbf{H} = 0$

Response of a medium

$$\mathbf{D}(\mathbf{r},t) = \epsilon_0 \mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t) \quad (5)$$

$$\mathbf{H}(\mathbf{r},t) = \frac{1}{\mu_0}\mathbf{B}(\mathbf{r},t) - \mathbf{M}(\mathbf{r},t) \quad (6)$$

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Wave equations

$$\nabla\times\nabla\times\mathbf{E}+\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = -\mu_o\frac{\partial}{\partial t}\left(\mathbf{j}+\frac{\partial\mathbf{P}}{\partial t}+\nabla\times\mathbf{M}\right)$$

$$\nabla\times\nabla\times\mathbf{H}+\frac{1}{c^2}\frac{\partial^2\mathbf{H}}{\partial t^2} = \nabla\times\mathbf{j}+\nabla\times\frac{\partial\mathbf{P}}{\partial t}+\mu_o\frac{\partial\mathbf{M}}{\partial t}\,.$$

Obtained by applying $\nabla\times$
to the Maxwell curl equations (1) and (2)
and using the material equations

c = $(\varepsilon_o\mu_o)^{-1/2}$ speed of light in vacuum

The source of the fields is a current density:

$$\mathbf{j}_t = \mathbf{j}_s+\mathbf{j}_c+\frac{\partial\mathbf{P}}{\partial t}+\nabla\times\mathbf{M}$$

source current density
conduction current density
polarization current density
magnetization current density

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Response of matter to e.m. fields

for linear, isotropic, non-dispersive media

$$\mathbf{D} = \varepsilon_0\varepsilon\mathbf{E} \qquad \mathbf{P} = \varepsilon_0\chi_e\mathbf{E}$$
$$\mathbf{B} = \mu_0\mu\mathbf{H} \qquad \mathbf{M} = \chi_m\mathbf{H}$$
$$\mathbf{j}_c = \sigma\mathbf{E}$$

- Higher order terms for nonlinear media
- Tensors for anisotriopic media
- Functions of space for inhomogeneous media

temporal dispersion: material parameters are functions of the frequency
spatial dispersion: material parameters are convolutions over space
→ non-local medium

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Time-harmonic fields

Separation of the wave equations → harmonic differential eq.

$$\mathbf{E}(\mathbf{r},t) \;=\; \mathrm{Re}\{\mathbf{E}(\mathbf{r})\,\mathrm{e}^{-i\omega t}\} \;=\; \frac{1}{2}\left[\mathbf{E}(\mathbf{r})\,\mathrm{e}^{-i\omega t} + \mathbf{E}^{*}(\mathbf{r})\,\mathrm{e}^{i\omega t}\right]$$

$$\mathbf{E}(\mathbf{r},t) \;=\; \mathrm{Re}\{\mathbf{E}(\mathbf{r})\}\cos\omega t + \mathrm{Im}\{\mathbf{E}(\mathbf{r})\}\sin\omega t \;=\; |\mathbf{E}(\mathbf{r})|\cos\left[\omega t + \varphi(\mathbf{r})\right]$$

Maxwell's equations (1) – (4) then become:

$$\begin{aligned}
\nabla\times\mathbf{E}(\mathbf{r}) &= i\omega\mathbf{B}(\mathbf{r})\,,\\
\nabla\times\mathbf{H}(\mathbf{r}) &= -i\omega\mathbf{D}(\mathbf{r}) + \mathbf{j}(\mathbf{r})\,,\\
\nabla\cdot\mathbf{D}(\mathbf{r}) &= \rho(\mathbf{r})\,,\\
\nabla\cdot\mathbf{B}(\mathbf{r}) &= 0\,,
\end{aligned}$$

complex, frequency dependent field amplitudes

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Complex dielectric constant

Constitutive relations

$$\begin{aligned}
\mathbf{D} &= \varepsilon_0\varepsilon\mathbf{E} & \mathbf{P} &= \varepsilon_0\chi_e\mathbf{E}\\
\mathbf{B} &= \mu_0\mu\mathbf{H} & \mathbf{M} &= \chi_m\mathbf{H}\\
\mathbf{j}_c &= \sigma\mathbf{E}
\end{aligned}$$

Maxwell:

$$\begin{aligned}
\nabla\times\mathbf{E}(\mathbf{r}) &= i\omega\mathbf{B}(\mathbf{r})\,,\\
\nabla\times\mathbf{H}(\mathbf{r}) &= -i\omega\mathbf{D}(\mathbf{r}) + \mathbf{j}(\mathbf{r})\,,\\
\nabla\cdot\mathbf{D}(\mathbf{r}) &= \rho(\mathbf{r})\,,\\
\nabla\cdot\mathbf{B}(\mathbf{r}) &= 0\,,
\end{aligned}$$

μ^{-1} $\nabla\times$

ε

$$\nabla\times\mu^{-1}\nabla\times\mathbf{E} - \frac{\omega^2}{c^2}\left[\varepsilon + i\sigma/(\omega\varepsilon_o)\right]\mathbf{E} = i\omega\mu_o\mathbf{j}_s$$

Dielectric function is a complex quantity

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With the new ε

$$\nabla \times \mu^{-1} \nabla \times \mathbf{E} - k_o^2 \varepsilon \, \mathbf{E} = i\omega\mu_o \mathbf{j}_s \,,$$

$$\nabla \times \varepsilon^{-1} \nabla \times \mathbf{H} - k_o^2 \mu \, \mathbf{H} = \nabla \times \; \varepsilon^{-1} \mathbf{j}_s$$

in linear, isotropic, but **inhomogeneous** media

$$k_o = \omega / c$$

Piecewise homogeneous media

Inhomogeneity is confined to the boundary between isotropic media!
→ Find solutions in each isotropic area and connect those via boundary conditions.

$$(\nabla^2 + k_i^2) \, \mathbf{E}_i = -i\omega\mu_o\mu_i \, \mathbf{j}_i + \frac{\nabla \rho_i}{\varepsilon_o \varepsilon_i}$$

$$(\nabla^2 + k_i^2) \, \mathbf{H}_i = -\nabla \times \mathbf{j}_i \,,$$

inhomogeneous vector Helmholtz equations

$$k_i = (\omega/c) \sqrt{\mu_i \varepsilon_i}$$

$\mathbf{j}_i, \; \rho_i$ sources in the domain D_i

Derivation: $\nabla \times \nabla \times = -\nabla^2 + \nabla\nabla \cdot$

$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t)$$

Boundary conditions

integral form of Maxwell´s equations (1) – (4)

Faraday's law

$$\int_{\partial S} \mathbf{E}(\mathbf{r},t) \cdot d\mathbf{s} \;=\; -\int_S \frac{\partial}{\partial t}\mathbf{B}(\mathbf{r},t) \cdot \mathbf{n}_s \; da \; ,$$

$$\int_{\partial S} \mathbf{H}(\mathbf{r},t) \cdot d\mathbf{s} \;=\; \int_S \left[\mathbf{j}(\mathbf{r},t) + \frac{\partial}{\partial t}\mathbf{D}(\mathbf{r},t)\right] \cdot \mathbf{n}_s \; da \; ,$$

$$\int_{\partial V} \mathbf{D}(\mathbf{r},t) \cdot \mathbf{n}_s \; da \;=\; \int_V \rho(\mathbf{r},t) \; dV \; ,$$

Gauss' law

$$\int_{\partial V} \mathbf{B}(\mathbf{r},t) \cdot \mathbf{n}_s \; da \;=\; 0 \; .$$

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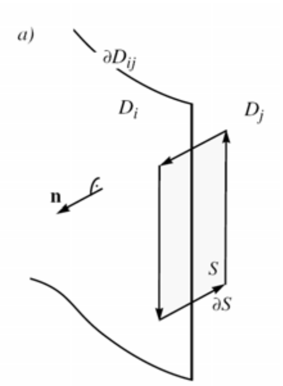
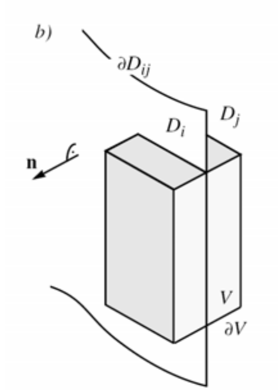
Boundary conditions



Parallel component of the electric field is continuous

$$\mathbf{n} \times (\mathbf{E}_i - \mathbf{E}_j) \;=\; \mathbf{0} \quad on \; \partial D_{ij} \; ,$$

$$\mathbf{n} \times (\mathbf{H}_i - \mathbf{H}_j) \;=\; \mathbf{K} \quad on \; \partial D_{ij} \; ,$$



Normal component of the dielectric displacement is continuous

$$\mathbf{n} \cdot (\mathbf{D}_i - \mathbf{D}_j) \;=\; \sigma \quad on \; \partial D_{ij}$$

$$\mathbf{n} \cdot (\mathbf{B}_i - \mathbf{B}_j) \;=\; 0 \quad on \; \partial D_{ij} \; .$$

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Poyntings theorem

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}, \qquad (1)$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} + \mathbf{j}(\mathbf{r},t), \qquad (2)$$

$\mathbf{H}\cdot$(1) - $\mathbf{E}\cdot$(2)

$$\mathbf{H}\cdot(\nabla\times\mathbf{E}) - \mathbf{E}\cdot(\nabla\times\mathbf{H}) = -\mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t} - \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t} - \mathbf{j}\cdot\mathbf{E}$$

$$= \nabla\cdot(\mathbf{E}\times\mathbf{H})$$

integrating both
sides over space and applying Gauss' theorem

$$\int_{\partial V} (\mathbf{E}\times\mathbf{H})\cdot\mathbf{n}\, da = -\int_V \left[\mathbf{H}\cdot\frac{\partial \mathbf{B}}{\partial t} + \mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}\cdot\mathbf{E}\right] dV$$

$$\mathbf{D}(\mathbf{r},t) = \varepsilon_o \mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t),$$

$$\mathbf{H}(\mathbf{r},t) = \mu_o^{-1}\mathbf{B}(\mathbf{r},t) - \mathbf{M}(\mathbf{r},t)$$

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Energy conservation

Flow of e.m. energy in or out of V

time rate of change of electromagnetic energy inside V

$$\int_{\partial V} (\mathbf{E}\times\mathbf{H})\cdot\mathbf{n}\, da + \frac{1}{2}\frac{\partial}{\partial t}\int_V \left[\mathbf{D}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{H}\right] dV =$$

$$-\int_V \mathbf{j}\cdot\mathbf{E}\, dV$$

for linear media

Energy dissipation in V
= Ohmic losses

$$\mathbf{S} = (\mathbf{E}\times\mathbf{H})$$

energy flux density,
Poynting vector

time- averaged
Poynting vector

$$\langle\mathbf{S}\rangle = \frac{1}{2}\mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\}$$

$$W = \frac{1}{2}\left[\mathbf{D}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{H}\right]$$

density of electromagnetic energy

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Dyadic Green´s function

The dyadic Green´s function defines the electric field at the point r generated by a radiating electric dipole source at r´

$$\mathbf{E}(\mathbf{r}) = \omega^2 \mu_o \mu \, \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') \, \boldsymbol{\mu}$$

Some basics:

$$\mathcal{L}\,\mathbf{A}(\mathbf{r}) = \mathbf{B}(\mathbf{r})$$

source function

unknown system response

complete solution = solution of the homogeneous problem + particular inhomogeneous solution

Assume homogeneous solution is known – how to find a particular inhomogeneous solution?

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Dyadic Green´s function

Consider special inhomogeneity $\delta(\mathbf{r}-\mathbf{r}')$

$$\mathcal{L}\,\mathbf{G}_i(\mathbf{r},\mathbf{r}') = \mathbf{n}_i\,\delta(\mathbf{r}-\mathbf{r}') \qquad (i{=}x,y,z)$$

oder $\mathcal{L}\,\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \overleftrightarrow{\mathbf{I}}\,\delta(\mathbf{r}-\mathbf{r}')$

$\mathcal{L}$ acts on each column of $\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ separately and $\overleftrightarrow{\mathbf{I}}$ is the unit dyad.

Once $\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ is known:

$$\int_V \mathcal{L}\,\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')\,\mathbf{B}(\mathbf{r}')\,dV' = \int_V \mathbf{B}(\mathbf{r}')\,\delta(\mathbf{r}-\mathbf{r}')\,dV'$$

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Dyadic Green´s function

$$\mathcal{L}\,\mathbf{A}(\mathbf{r}) \;=\; \int_V \mathcal{L}\,\overleftrightarrow{\mathbf{G}}\,(\mathbf{r},\mathbf{r}')\,\mathbf{B}(\mathbf{r}')\;dV' \quad \text{mit} \quad \mathcal{L}\,\mathbf{A}(\mathbf{r}) \;=\; \mathbf{B}(\mathbf{r})$$

$$\rightarrow \mathbf{A}(\mathbf{r}) \;=\; \int_V \overleftrightarrow{\mathbf{G}}\,(\mathbf{r},\mathbf{r}')\,\mathbf{B}(\mathbf{r}')\;dV'$$

What is $\overleftrightarrow{\mathbf{G}}\,(\mathbf{r},\mathbf{r}')$ for the electric field?

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Dyadic Green´s function

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= i\omega\mathbf{A}(\mathbf{r}) - \nabla\phi(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}) &= \frac{1}{\mu_o\mu}\nabla\times\mathbf{A}(\mathbf{r})\;. \end{aligned}$$

plugged into $\nabla\times\mathbf{H}(\mathbf{r},t) \;=\; \dfrac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t} + \mathbf{j}(\mathbf{r},t)$

using $\mathbf{D}=\varepsilon_o\varepsilon\mathbf{E}$

$$\rightarrow \nabla\times\nabla\times\mathbf{A}(\mathbf{r}) \;=\; \mu_o\mu\mathbf{j}(\mathbf{r}) - i\omega\mu_o\mu\varepsilon_o\varepsilon\left[i\omega\mathbf{A}(\mathbf{r}) - \nabla\phi(\mathbf{r})\right]$$

Lorenz gauge: $\nabla\cdot\mathbf{A}(\mathbf{r}) \;=\; i\omega\mu_o\mu\varepsilon_o\varepsilon\phi(\mathbf{r})$

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Dyadic Green´s function

using again $\nabla\times\nabla\times = -\nabla^2 + \nabla\nabla\cdot$

$$\rightarrow \quad \left[\nabla^2 + k^2\right]\mathbf{A}(\mathbf{r}) = -\mu_o\mu\mathbf{j}(\mathbf{r})$$
$$\left[\nabla^2 + k^2\right]\phi(\mathbf{r}) = -\rho(\mathbf{r})/\varepsilon_o\varepsilon$$

four scalar Helmholtz equations of the form

$$\left[\nabla^2 + k^2\right] f(\mathbf{r}) = -g(\mathbf{r})$$

to obtain the scalar Green's function for the Helmholtz operator:

$$\left[\nabla^2 + k^2\right] G_o(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}')$$

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Dyadic Green´s function

$$G_o(\mathbf{r},\mathbf{r}') = \frac{e^{\pm ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

free-space Green´s function of the Helmholtz operator

$$\rightarrow \mathbf{A}(\mathbf{r}) = \mu_o\mu\int_V \mathbf{j}(\mathbf{r}')\,G_o(\mathbf{r},\mathbf{r}')\,dV'$$

How about the fields?

for the fields the Greens function must be a tensor since a current element in x direction causes fields in x, y, and z direction!

$$\mathbf{E}(\mathbf{r}) = i\omega\mathbf{A}(\mathbf{r}) - \nabla\phi(\mathbf{r})$$ together with the Lorenz gauge

$$\rightarrow \mathbf{E}(\mathbf{r}) = i\omega\left[1 + \frac{1}{k^2}\nabla\nabla\cdot\right]\mathbf{A}(\mathbf{r})$$

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Dyadic Green´s function

The vector potential originating from a point source current

$$\mathbf{j} = (i\omega\mu_o)^{-1}\delta(\mathbf{r}-\mathbf{r}')\mathbf{n}_x$$

because of $\mathbf{A}(\mathbf{r}) = \mu_o\mu \int_V \mathbf{j}(\mathbf{r}')\, G_o(\mathbf{r},\mathbf{r}')\, dV'$ is:

$$\mathbf{A}(\mathbf{r}) = (i\omega)^{-1}\, G_o(\mathbf{r},\mathbf{r}')\, \mathbf{n}_x$$

putting this into $\mathbf{E}(\mathbf{r}) = i\omega \left[1 + \frac{1}{k^2}\nabla\nabla\cdot\right] \mathbf{A}(\mathbf{r})$

point source current!

$$\mathbf{G}_x(\mathbf{r},\mathbf{r}') = \left[1 + \frac{1}{k^2}\nabla\nabla\cdot\right] G_o(\mathbf{r},\mathbf{r}')\, \mathbf{n}_x$$

$$\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left[\overleftrightarrow{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\right] G_o(\mathbf{r},\mathbf{r}')$$

$$\nabla\cdot[G_o\,\overleftrightarrow{\mathbf{I}}] = \nabla G_o$$

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Dyadic Green´s function

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_o(\mathbf{r}) + i\omega\mu_o\mu \int_V \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')\,\mathbf{j}(\mathbf{r}')\, dV' \qquad \mathbf{r}\notin V$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_o(\mathbf{r}) + \int_V \left[\nabla\times\overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')\right]\mathbf{j}(\mathbf{r}')\, dV' \qquad \mathbf{r}\notin V$$

Volume integral equations

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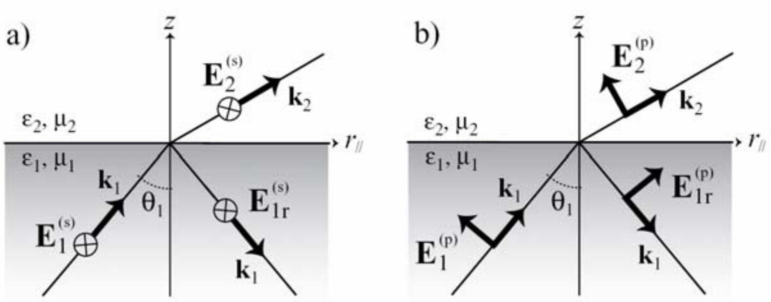
Evanescent fields

evanescere: latin. Has meanings like *vanishing from notice* or *imperceptible*.

$\mathbf{E}e^{i(\mathbf{kr}-\omega t)}$ plane waves with one component of the wave vector imaginary.

This never occurs in a homogeneous medium!

Simplest inhomogeneity: plane interface between two media



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Evanescent fields

$$\mathbf{E}_2 = \begin{bmatrix} -\mathbf{E}_1^{(p)}\, t^p(k_x)\, k_{z_2}/k_2 \\ \mathbf{E}_1^{(s)}\, t^s(k_x) \\ \mathbf{E}_1^{(p)}\, t^p(k_x)\, k_x/k_2 \end{bmatrix} \mathrm{e}^{ik_x x + ik_{z_2} z}$$

with the Fresnel reflection and transmission coefficients

$$r^s(k_x,k_y) = \frac{\mu_2 k_{z_1} - \mu_1 k_{z_2}}{\mu_2 k_{z_1} + \mu_1 k_{z_2}}, \qquad r^p(k_x,k_y) = \frac{\varepsilon_2 k_{z_1} - \varepsilon_1 k_{z_2}}{\varepsilon_2 k_{z_1} + \varepsilon_1 k_{z_2}}$$

$$t^s(k_x,k_y) = \frac{2\mu_2 k_{z_1}}{\mu_2 k_{z_1} + \mu_1 k_{z_2}}, \qquad t^p(k_x,k_y) = \frac{2\varepsilon_2 k_{z_1}}{\varepsilon_2 k_{z_1} + \varepsilon_1 k_{z_2}} \sqrt{\frac{\mu_2 \varepsilon_1}{\mu_1 \varepsilon_2}}$$

$$k_{z_1} = \sqrt{k_1^2 - (k_x^2 + k_y^2)}., \qquad k_{z_2} = \sqrt{k_2^2 - (k_x^2 + k_y^2)}.$$

$$k_{z_1} = k_1 \sqrt{1 - \sin^2\theta_1}, \qquad k_{z_2} = k_2 \sqrt{1 - \tilde{n}^2 \sin^2\theta_1}$$

$$\tilde{n} = \frac{\sqrt{\varepsilon_1 \mu_1}}{\sqrt{\varepsilon_2 \mu_2}}$$

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Evanescent fields

with increasing θ_1 the argument of the square root in the expression of k_{z2} gets smaller and smaller and eventually becomes negative. The critical angle θ_c can be defined by the condition

$$\left[1 - \tilde{n}^2 \sin^2\theta_1\right] = 0 \quad \rightarrow \quad \theta_c = \arcsin\left[1/\tilde{n}\right]$$

$$\varepsilon_2 = 1, \ \varepsilon_1 = 2.25 \rightarrow \quad \theta_c = 41.8°$$

For $\theta_1 > \theta_c$, k_{z_2} becomes imaginary

$$\mathbf{E}_2 = \begin{bmatrix} -i\mathbf{E}_1^{(p)}\, t^p(\theta_1)\sqrt{\tilde{n}^2 \sin^2\theta_1 - 1} \\ \mathbf{E}_1^{(s)}\, t^p(\theta_1) \\ \mathbf{E}_1^{(p)}\, t^p(\theta_1)\, \tilde{n}\sin\theta_1 \end{bmatrix} \mathrm{e}^{i \sin\theta_1 k_1 x}\, \mathrm{e}^{-\gamma z}$$

$$\gamma = k_1 \sqrt{\tilde{n}^2 \sin^2\theta_1 - 1}$$

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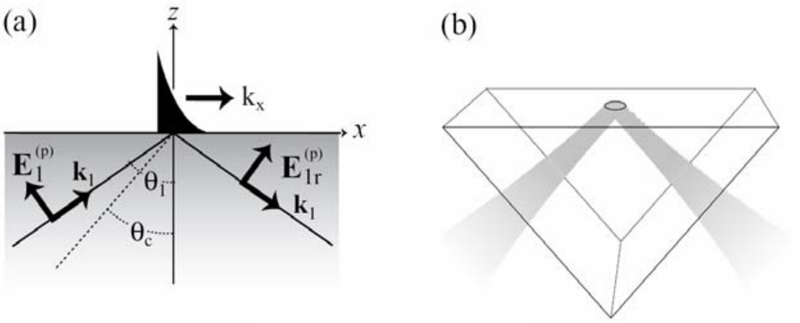
Evanescent fields



for $\theta_i = 45°$ and $\varepsilon_2 = 1, \ \varepsilon_1 = 2.25$

$\rightarrow \gamma = 2.22/\lambda$

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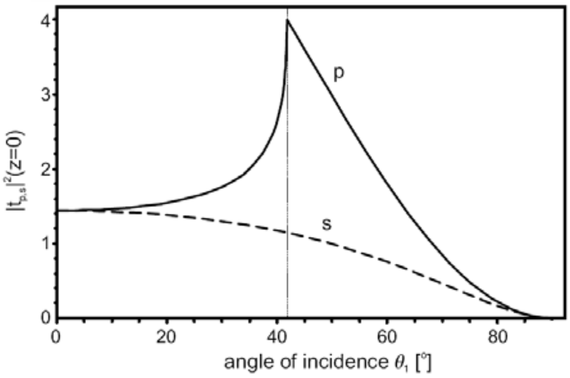
Evanescent fields

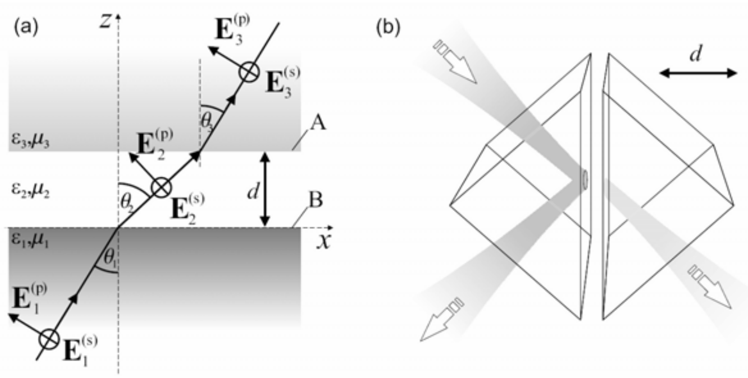
$$|\mathbf{E}_2(z=0)|/|\mathbf{E}_1(z=0)|$$



Energy transport by evanescent waves?

Mearsurement of evanescent fields

Frustrated TIR



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Frustrated TIR

Perpendicular component of the wave vector in each medium:

$$k_{j,z} = \left(\epsilon_j \mu_j k^2 - k_{\|}^2\right)^{1/2} = k\left(\epsilon_j \mu_j - \epsilon_1 \mu_1 \sin^2\theta_1\right)^{1/2}, \quad j \in \{1,2,3\}$$

(exploit the fact that the parallel component is conserved!)

Distinguish 3 cases:

1.: $\theta_1 < \arcsin(n_2/n_1)$ or $k_{\|} < n_2 k$

2.: $\arcsin(n_2/n_1) < \theta_1 < \arcsin(n_3/n_1)$ or $n_2 k < k_{\|}$

3.: $\theta_1 > \arcsin(n_3/n_1)$ or $k_{\|} > n_3 k$

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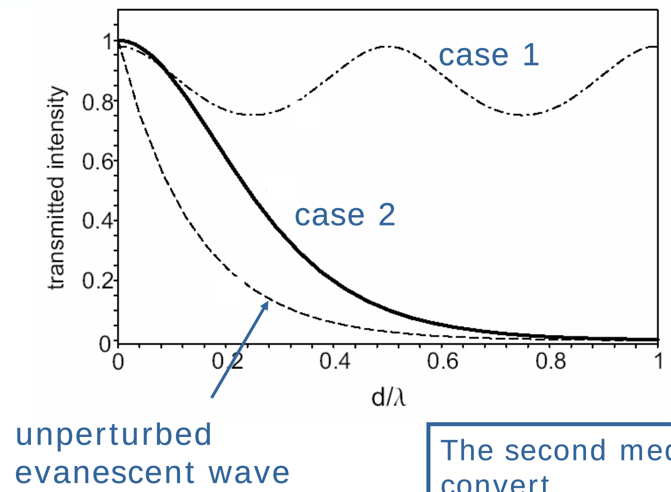
Transmitted far-field intensity



The second medium can convert evanescent fields into propagating fields!

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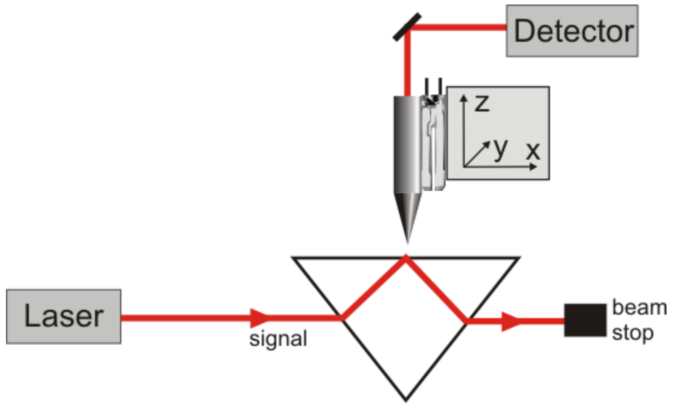
STOM

Scanning tunneling optical microscope



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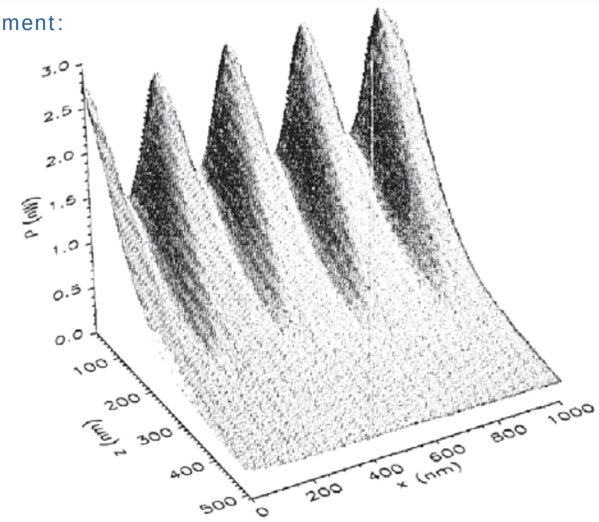
Evanescent fields

Experiment:



A. Meixner, M. Bopp, and G. Tarrach, "Direct Measurement of Standing Evanescent Waves with a Photon Scanning Tunneling Microscope," Appl. Opt. **33**, 7995 (1994).

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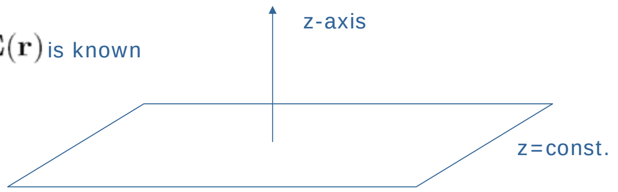
Angular spectrum representation of optical fields

- mathematical technique to describe optical fields in homogeneous media
- Optical fields are described as a superposition of plane waves and evanescent waves which are physically intuitive solutions of Maxwell's equations.

$\mathbf{E}(\mathbf{r})$ is known



$$\hat{\mathbf{E}}(k_x, k_y; z) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \mathbf{E}(x, y, z)\, e^{-i[k_x x + k_y y]}\, dx\, dy$$

two-dimensional Fourier transform of the field

k_x, k_y spatial frequencies

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inverse Fourier transform:

$$\mathbf{E}(x, y, z) = \iint_{-\infty}^{\infty} \hat{\mathbf{E}}(k_x, k_y; z)\, e^{i[k_x x + k_y y]}\, dk_x\, dk_y$$

the Fourier integrals hold separately for each vector component

homogeneous, isotropic, linear and source-free medium:

$$(\nabla^2 + k^2)\, \mathbf{E}(\mathbf{r}) = 0$$

$k = (\omega/c)\, n$

$n = \sqrt{\mu\varepsilon}$

$$\hat{\mathbf{E}}(k_x, k_y; z) = \hat{\mathbf{E}}(k_x, k_y; 0)\, e^{\pm i k_z z}$$

$$k_z \equiv \sqrt{(k^2 - k_x^2 - k_y^2)}$$

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